THE PHYSICAL SOCIETY OF LONDON.

PROCEEDINGS.

VOLUME XXXII.—PART V.

AUGUST 15, 1920.

Price to Non-Fellows, **4s.** *net, post free* **4/3.**

Annual Subscription, **20/-** *post free, payable in advance,*

Published Bi-Monthly from December to August.

LONDON:
FLEETWAY PRESS, LTD.,
1, 2 AND 3, SALISBURY COURT, FLEET STREET

1920.

THE PHYSICAL SOCIETY OF LONDON.

1920-21.

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PROCEEDINGS

AT THE

MEETINGS OF THE PHYSICAL SOCIETY

OF LONDON.

SESSION 1919–1920.

October 24, 1919.

Meeting held at Imperial College of Science.

Prof. C. H. Lees, F.R.S., President, in the Chair.

The following Papers were read :—

1. "The Effect of Pressure and Temperature on a Meter for Measuring the Rate of Flow of a Gas." By Dr. N. W. McLachlan.

2. "A Cheap and Simple Micro-Balance." By Capt. J. H. Shaxby.

3. "The Resolution of a Curve into a Number of Exponential Components." By J. W. T. Walsh.

November 14, 1919.

Meeting held at the Imperial College of Science.

Prof. C. H. Lees, F.R.S., President, in the Chair.

The following Papers were read :—

1. "On the Self-inductance of Single-layer Flat Coils." By S. Butterworth.

2. "An Experimental Method of Determining the Primary Current at Break in a Magneto." By Dr. N. W. McLachlan.

A New Form of Wehnelt Interrupter was shown by F. H. Newman.

November 28, 1919.

Meeting held at Imperial College of Science.

Prof. C. H. Lees, F.R.S., President, in the Chair.

A Discussion on Lubrication was held.

December 12, 1920.

Meeting held at Imperial College of Science.

Prof. C. H. Lees, F.R.S., President, in the Chair.

The following Papers were read :—

1.* "First Steps in the Experimental Analysis of a Galvanic Cell." By Prof. W. M. Coleman.

2. "Radiation from a Perfectly Diffusing Circular Disc." By J. W. T. Walsh.

3. "Testing of Thermionic Valves for Non-passage of Reverse Current at High Voltage." By Dr. N. W. McLachlan.

4. "On Recording and Reproducing Sounds by Means of Light." By Prof. A. O. Rankine.

* Read by Dr. D. Owen, in the absence of the Author.

January 7 and 8, 1920.

The Tenth Annual Exhibition of Scientific Apparatus was held in conjunction with the Optical Society.

Short Lectures were given on both days by

Prof. F. J. Cheshire, on "Some Polarisation Experiments," and

Prof. A. O. Rankine, on "The Use of Light in the Transmission and Reproduction of Speech."

January 23, 1920.

Meeting held at City and Guilds Technical College, Finsbury.

Prof. C. H. Lees, F.R.S., President, in the Chair.

The following Papers were read :—

1. "On Some Experiments in which Two Neighbouring Maintained Oscillatory Circuits Affect a Resonating Circuit." By Dr. J. H. Vincent.

2. "Measurement of the Chief Parameters of Triode Valves." By Prof. W. H. Eccles.

3. "Measurement of the Amplification of a Radio-Frequency Amplifier." By Mr. F. W. Jordan.

4. "Measurement of the Amplification Given by Triode Amplifiers at Audible and Radio Frequencies." By F. E. Smith and the late H. C. Napier.

The following were exhibited :—

(1) An Instrument for Indicating the Hardening Temperature of Steel. By the Hon. C. W. Stopford and C. R. Darling.

(2) A Thermo-electric Cell of Constant Voltage. By C. R. Darling.

February 13, 1920.

Annual General Meeting.

Held at Imperial College of Science.

Prof. C. H. LEES delivered the Presidential Address, taking as his subject "The Temperature of the Earth's Interior."

A Paper on "The Influence of Small Changes of Temperature on Atmospheric Refraction" was read by Sir ARTHUR SCHUSTER.

GENERAL BUSINESS.

It was moved by the PRESIDENT and seconded by Dr. BORNS that the reports of the Secretary and Treasurer be adopted. This was unanimously carried.

DRAFT REPORT OF COUNCIL.

In the year 1919, 13 ordinary Science Meetings and one Special General Meeting of the Society have been held. Of these the Special General Meeting and 12 ordinary Meetings were held at the Imperial College of Science, and one ordinary Meeting, that on July 11, was held at the National Physical Laboratory by invitation of Sir Richard Glazebrook, K.C.B., F.R.S. The average attendance at the Meetings was 53, as compared with 35 during the preceding year.

At a Special General Meeting held on June 13, certain proposals made by the Institute of Physics were recommended for adoption by the Physical Society as one of the participating Societies. These proposals which affect subscriptions and the cost of certain publications were adopted unanimously and will become operative when the Institute of Physics is fully established.

The ordinary Meeting on March 28 was devoted to a discussion on Metrology and its application to the Industries. It was very successful and was attended by 91 Fellows and Visitors.

On November 28 a discussion on Lubrication was held. There were numerous contributions to the discussion, and more than 100 Fellows and Visitors were in attendance.

There have been two resignations of Officers of the Society. Dr. Allen has been appointed to Edinburgh University and resigned the Secretaryship in June. Prof. S. W. J. Smith, a former Secretary and for many years Librarian of the Society, resigned the latter position on his appointment as Professor of Physics at Birmingham University. He is succeeded by Dr. Rankine as Librarian.

During the Session the Society arranged for the publication of a second edition of Prof. Eddington's Report on the "Relativity Theory of Gravitation," and for a Report by Prof. Fowler, F.R.A.S., F.R.S., on "Series in Spectra"

The number of Honorary Fellows on the roll on December 31, 1919, was 10, and the number of ordinary Fellows was 484, an increase of 39 on the number of last year. Fifty new Fellows and 7 Students have been elected, and 3 Fellows have resigned.

The Society has to mourn the loss by death of 8 Fellows, namely, Prof. Carey Foster, Prof. W. Watson, Sir William Crookes, Lord Rayleigh, Mr. D. Rintoul, Mr. F. J. Fry, Mr. W. R. Phelips, and S. D. Chalmers. Prof. Carey Foster was a past President and was one of the founders of the Society. Prof. W. Watson was for many years a Secretary of the Society and afterwards served as Librarian.

Report of the Treasurer.

The accounts for the past year show a great improvement in the financial position of the Society. The total income is £1,261. 2s. 7d., as against £1,055. 8s. 0d. for the previous year. This is due partly to the growth in membership leading to increased subscriptions, but in the main to the larger sale of publications. The amount realised by sales was £280. 16s. 10d., as against £148. 2s. 4d. in 1918.

On the other hand, more matter has been published in the "Proceedings," and, although no special publications have been issued during the year, the amount spent on printing has increased. Other items have not varied seriously, and thus there is a substantial balance of £273. 15s. 11d. of income over expenditure.

In the annual accounts hitherto it has been customary to include the cost of the publications over a complete volume (or session), which does not coincide with the financial year. The Council have now decided that it is preferable in all cases to include in the accounts the expenditure which has actually been incurred during the particular year, as distinct from the session, so that the expenditure should truly correspond with the revenue and with the other items of expenditure. Consequently there is a sum of £95. 6s. 0d., mainly expended on Part I. of Vol. XXXI. of the "Proceedings" (at the end of 1918), which does not appear in the accounts of either 1918 or 1919.

The Balance Sheet shows that the position in regard to arrears of subscriptions is increasingly unsatisfactory. The sum recovered during the past year has been much less than was hoped, and for this session a heavy reserve has been set against the sum due.

The investments, as in previous years, have been valued at market prices through the courtesy of the London, County, Westminster & Parr's Bank. They show a further depreciation in value. During the year under review the Exchequer Bonds and National War Bonds were converted into Funding Loan, and a further sum of £20 was invested in the latter to bring the total up to a round figure. The cash on deposit at the Bank has increased by £200.

The accounts as a whole show that the normal activities of the Society are not likely to suffer in the future through lack of income.

INCOME AND EXPENDITURE ACCOUNT.

From January 1st to December 31st, 1919.

Income.	£	s.	d.	£	s.	d.
Entrance Fees	49	7	0			
Subscriptions by Fellows	537	12	0			
,, Voluntary	16	16	0			
,, by Students	2	12	6			
,, Arrears paid	33	14	6			
,, Paid in Advance	41	17	6			
,, for "Science Abstracts" and Advance Proofs	19	5	0			
				701	4	6
Composition Fees				94	10	0
Dividends :—						
Furness Debenture Stock	11	3	10			
Midland Railway	28	5	0			
Metropolitan Board of Works	7	0	0			
Lancaster Corporation Stock	8	8	0			
New South Wales Stock	8	17	10			
London, Brighton & South Coast Railway	18	12	6			
Great Eastern Railway	14	0	0			
India $3\frac{1}{2}$% Stock	17	10	0			
Exchequer Bonds, 5%	14	4	4			
National War Bonds, 5%	3	12	2			
Funding Loan	6	12	9			
				138	6	5
Income Tax refunded				38	0	1
Interest on deposit account				8	4	9
Sales of Publications (Fleetway Press, Ltd.)				280	16	10
				£1,261	2	7

Expenditure.	£	s.	d.	£	s.	d.
"Science Abstracts"	282	12	0			
,, ,, Extra copies	4	4	0			
				286	16	0
Fleetway Press, Ltd. :—						
"Proceedings"	428	2	11			
Bulletin	50	18	8			
Distribution (Postage)	50	14	5			
General	52	0	6			
				581	16	6
Periodicals	1	15	0			
Reporting	47	5	8			
Refreshments and Attendance	21	10	9			
Petty Cash—						
Secretaries' Expenses	14	10	11			
Treasurer's Expenses	4	11	4			
Insurance	3	2	6			
Advertising	3	14	0			
Royal Asiatic Society	2	2	0			
Bank Charges		2	0			
				98	14	2
Donations—						
Conjoint Board of Scientific Societies	10	0	0			
Annual Tables of Constants	10	0	0			
				20	0	0
Balance, being excess of income over expenditure				273	15	11
				£1,261	2	7

W. R. COOPER, *Honorary Treasurer.*

Audited and found correct,

W. A. J. O'MEARA, G. W. O. HOWE, } *Honorary Auditors.*

February 5th, 1920.

BALANCE SHEET AT DECEMBER 31st, 1919.

Assets.

	£	s.	d.	£	s.	d.
Subscriptions in arrears	255	3	0			
Less reserve for subscriptions probably unrealisable	150	0	0			
				105	3	0
Investments (valued at Dec. 31):—						
£533 Furness 3 per cent. Debenture Stock	261	0	0			
£1,600 Midland Railway 2½ per cent. Perpetual Preference Stock	680	0	0			
£200 Metropolitan Board of Works 3½ per cent. Consolidated Stock	162	0	0			
£400 Lancaster Corporation 3 per cent. Redeemable Stock	224	0	0			
£254 2s. 9d. New South Wales 3½ per cent. Ordinary Stock	218	0	0			
£500 London, Brighton & South Coast Railway Ordinary Stock...	350	0	0			
£500 Great Eastern Railway 4 per cent. Debenture Stock	335	0	0			
£500 India 3½ per cent. Stock......	305	0	0			
£650 4% Funding Loan, 1960-90	494	0	0			
				3,029	0	0
Stock of Publications (Treasurer's valuation				250	0	0
Cash at Bank on Deposit				250	0	0
Cash at Bank, Current Account at Dec. 31	218	3	11			
Adjustment for outstanding cheques	121	5	0			
				96	18	11
Cash in hand Treasurer's Petty Cash)				1	5	5
				£3,732	7	4

W. R. COOPER, *Honorary Treasurer.*

Liabilities.

	£	s.	d.
Life Compositions	1,946	10	0
Balance, General Fund	1,785	17	4
	£3,732	7	4

Audited and found correct,

W. A. J. O'MEARA, G. W. O. HOWE, } *Honorary Auditors.*

February 5th, 1920.

LIFE COMPOSITION FUND AT DECEMBER 31ST, 1919.

	£	s.	d.
145 Fellows paid £10	1,450	0	0
3 Fellows paid £15	45	0	0
8 Fellows paid £21	168	0	0
9 Fellows paid £31. 10s.	283	10	0
	£1,946	10	0

Audited and found correct,

W. R. COOPER, *Honorary Treasurer.*

February 5th, 1920.

W. A. J. O'MEARA, } *Honorary Auditors.*
G. W. O. HOWE,

On behalf of the Council, the President moved: "That this General Meeting of the Physical Society of London approves of the foundation of an International Union of Physics."

This was seconded by Mr. GUILD and carried unanimously.

The election of Officers and Council for the ensuing year resulted as follows:—

President.—Sir W. H. BRAGG, C.B.E., M.A., F.R.S.

Vice-Presidents, who have filled the office of President.—Prof. R. B. CLIFTON, M.A., F.R.S.; Prof. A. W. REINOLD, C.B., M.A., F.R.S.; Sir W. DE W. ABNEY, R.E., K.C.B., D.C.L., F.R.S.; Sir OLIVER J. LODGE, D.Sc., F.R.S.; Sir RICHARD GLAZEBROOK, K.C.B., D.Sc., F.R.S.; Prof. J. PERRY, D.Sc., F.R.S.; C. CHREE, Sc.D., LL.D., F.R.S.; Prof. H. L. CALLENDAR, M.A., LL.D., F.R.S.; Sir ARTHUR SCHUSTER, Ph.D., Scd., F.R.S.; Sir J. J. THOMSON, O.M., D.Sc., F.R.S.; Prof. C. VERNON BOYS, F.R.S.; Prof. C. H. LEES, D.Sc., F.R.S.

Vice-Presidents.—H. S. ALLEN, M.A., D.Sc.; Prof. W. ECCLES, D.Sc.; Prof. A. S. EDDINGTON, M.A., M.Sc., F.R.S.; R. S. WILLOWS, M.A., D.Sc.

Secretaries.—D. OWEN, B.A., D.Sc.; F. E. SMITH, O.B.E., F.R.S.

Foreign Secretary.—Sir ARTHUR SCHUSTER, Ph.D., ScD., F.R.S.

Treasurer.—W. R. COOPER, M.A., B.Sc.

Librarian.—A. O. RANKINE, D.Sc.

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The customary votes of thanks concluded the business of the meeting.

February 27, 1920.

Meeting held at Imperial College of Science.

Prof. W. H. Bragg, F.R.S., President, in the Chair.

The following Papers were read :—

1. " On Balancing Errors of Different Orders." By T. Smith.

2. " Notes on the Testing of Bars of Magnet Steel." By Dr N. W. McLachlan.

3. " On the Forces Acting on Heated Metal Foil." By G. D. West.

Miss N. Hosali exhibited a number of Crystal Models.

March 12, 1920.

Meeting held at Imperial College of Science.

Prof. W. H. Bragg, F.R.S., President, in the Chair.

The following Papers were read :—

1. " The Absorption of Gases in a Discharge Tube." By F. W. Newman.

2. " A Directional Hot-wire Anemometer of High Sensitivity, especially Applicable to the Investigation of Slow Rates of Flow of Gases." By J. S. G. Thomas.

3. " Experiments with a New Micro-Balance." By Dr. Hans Pettersson.

March 26, 1920.

Meeting held at the Imperial College of Science.

Prof. W. H. Bragg, F.R.S., President, in the Chair.

A Discussion on Einstein's Theory of Relativity was held.

April 23, 1920.

Meeting held at Imperial College of Science.

Prof. W. H. Bragg, F.R.S., President, in the Chair.

The Fifth Guthrie Lecture was delivered by Charles Edward Guillaume; *subject*—

"The Anomaly of the Nickel Iron Alloys; its Causes and Applications."

May 14, 1920.

Meeting held at Imperial College of Science.

Prof. W. H. Bragg, F.R.S., President, in the Chair.

A Demonstration of Experiments on the Thermionic Properties of Hot Filaments was given by Dr. F. Ll. Hopwood.

The following Papers were read:

1. "A Modified Theory of the Crookes Radiometer." By G. D. West.

2.*"On the Magnetic Properties of Silicon Iron (Stalloy) in Alternating Magnetic Fields of Low Value." By A. Campbell.

3. "On Tracing Rays through an Optical System." By T. Smith.

* Read by Mr. D. W. Dye, in the absence of the Author.

May 28, 1920.

Meeting held at Imperial College of Science.

Prof. W. H. Bragg, F.R.S., President, in the Chair.

A Discussion on X-Ray spectra was held.

June 11, 1920.

Meeting held at Imperial College of Science.

Prof. W. H. BRAGG, F.R.S., President, in the Chair.

The following Papers were read :—

1. " Radiation and Convection from Heated Surfaces." By Dr. T. BARRATT and Mr. A. J. SCOTT.

2. " An Electrical Hot-wire Inclinometer." By J. S. G. THOMAS.

3. " Convective Cooling and the Theory of Dimensions." By L. F. RICHARDSON.

4.*" The Radiation from a Perfectly Diffusing Circular Disc." By J. W. T. WALSH.

* Taken as read in the absence of the Author.

June 25, 1920.

Meeting held at Imperial College of Science.

Prof. W. H. BRAGG, F.R.S., President, in the Chair.

The following Papers were read :—

1. " The Origin of the Elements." By Dr. J. H. VINCENT.

2. " The Construction of Thermo-couples by Electro-deposition." By W. HAMILTON WILSON and Miss T. D. EPPS.

3. " The Use of Vacuum Arcs for Interferometry." By J. GUILD.

4. *" The Maintenance of a Vibrating System by Means of a Triode Valve." By S. BUTTERWORTH.

* Read, in the absence of the Author, by Mr. R. L. SMITH-ROSE.

XXV. *On the Mechanical Equilibrium of a Sphere of Gravitating Fluid.* *By* CHARLES H. LEES, *D.Sc., F.R.S.* (*Part of the Presidential Address to the Physical Society, February* 13, 1920.)

1. It is now nearly 60 years since, in a letter to Joule which was read at the meeting of the Manchester Literary and Philosophical Society on January 21, 1862, Lord Kelvin showed that convection currents in the earth's atmosphere would cause the average temperature of horizontal layers to decrease with height according to a law he named that of "convective equilibrium."*

If the rate of decrease of the temperature θ with height h above the surface of the earth is given by the equation

$$-\frac{d\theta}{dh}=\frac{\gamma-1}{\gamma}\cdot\frac{g}{R}, \quad \ldots \ldots \quad (1{\cdot}1)$$

where γ is the ratio of the specific heats of air at constant pressure and volume respectively, g is the gravitational acceleration, and R the gas constant of the gas equation $pv=R\theta$, where v is the volume of 1 gram of air at the point, the atmosphere is in "convective equilibrium." If the rate of decrease of temperature with height exceeds this value, the atmosphere is unstable; if it is less, the equilibrium is stable; if it has this value, the equilibrium is neutral.

With the values of R and γ for dry air, and of g for points not far from the surface, we find that the equilibrium is stable if the decrease of temperature with altitude is less than 1°C. per 100 metres. The difference between this result and 0·6°C. per 100 metres, found from observation, was shown by Kelvin to be due to the air not being dry, and, by taking into account the heat developed by the condensation of moisture as saturated air is cooled, he showed that the theoretical value of the rate of decrease was identical with that given by observation.†

* Kelvin, Proc. Manc. Lit. and Phil. Soc., 3rd series, II., p. 125 (1862). Abbe, in his "Mechanics of the Earth's Atmosphere" (Washington (1891), p. 172), states that the law of decrease of temperature with height was known to Espy and Henry in the United States, but he gives no references.

† Kelvin, *loc. cit.*

Kelvin's work does not appear to have been well known, for Peslin,* Reye† and Hann‡ all dealt subsequently with the problem for dry air without referring to it.

For meteorological purposes, the effects of the presence of water vapour in the atmosphere on the vertical distribution of temperature have been traced more in detail by Hertz,§ Bezold,|| Neuhoff ¶ and others.

2. In cosmological directions, Kelvin's results for a dry gas in a gravitational field of constant strength have been extended to spherical gaseous masses under their own gravitation by Homer Lane,** by Kelvin†† and by Jeans.‡‡

In these extensions the gaseous medium is assumed to be subject to the perfect gas equation $pv=\frac{R}{m}\cdot\theta$, where p is the pressure, v the specific volume, θ the absolute temperature, R is the gas constant and m is the molecular weight of the gas, or, if the gas is a mixture, m may be supposed defined by the equation.§§

If for the gas when adiabatically expanded $p=A\rho^{\gamma}$, where γ is the ratio of the specific heats at constant pressure and constant volume, ρ is the density and A is called by Kelvin the "adiabatic constant," for neutral or convective equilibrium A is constant throughout the gas at any instant, but diminishes as time goes on,|||| while for stable equilibrium A must increase from the centre of the gas to the circumference.¶¶

3. The inconvenience of the restriction of these propositions to media which satisfy the perfect gas equation has been felt by many workers in this field, and it is the object of the present note to show that a criterion for the stable equilibrium of any gravitating fluid or mixture of fluids may be established. By fluid is understood a medium in which the pressure is of the

* Peslin, Bull. Assoc. Scientif. de France, III., p. 299 (1868).

† Reye, Die Wirbelsturme, Hannover (1872).

‡ Hann, Zeit. Oester. Ges. f. Met., IX., pp. 321, 337 (1874).

§ Hertz, Met. Zeitsch., I., p. 421 (1884).

|| Bezold, Sitz. Ber. K. Akad., Berlin, XLVI., p. 485, 1,189.

¶ Neuhoff, Abh. K.P. Met. Inst., Berlin, I. (1900).

** Homer Lane, "American Journal of Science," LIII., p. 57 (1870).

†† Kelvin, "Phil. Mag.," XXII., p. 287 (1887), XV., p. 687, and XVI., p. 1 (1908).

‡‡ Jeans' "Cosmogony and Stellar Dynamics," Cambridge (1919), p. 191.

§§ Jeans, *loc. cit.*, p. 191.

|||| Kelvin, "Phil. Mag.," XVI., p. 5 (1908).

¶¶ This fact, though implied in the reasoning of Lane and Kelvin, does not appear to have been mentioned by them. It is stated clearly by Jeans, *loc. cit.*, pp. 193-4.

hydrostatic type—that is, has the same value in all directions about a point of the medium. Solids under great pressure probably come within this category along with liquids and gases.

4. If in a fluid we draw a surface everywhere perpendicular to the direction of the gravitational force at points in it, that surface is by Clairault a surface both of equal pressure and of equal density, whether the medium be homogeneous or not. If the gravitational force has the same value at all points of such a surface, the surface itself must be spherical, and the surfaces of equal gravitational force, equal pressure and equal density are all concentric spheres.

Let r be the radius of one of these spheres, p the pressure over it, and ρ the density of the medium at it. Then, if $r+dr$, $p+dp$ and $\rho+d\rho$ be the corresponding quantities at a consecutive concentric sphere, we have for mechanical equilibrium

$$dp=-g\rho\,.\,dr \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (4{\cdot}1)$$

where g is the value of the gravitational acceleration at the spherical shell. This condition does not require the composition of each shell to be identical throughout its volume, but merely its density the same throughout. Nor does it imply that ρ is a continuous function of r; abrupt changes of density with radius are not excluded.

5. If a small volume, say 1 cc. of the fluid of density ρ, be removed, and be replaced by an equal volume of a medium density, ρ', the gravitational force on the substituted matter will be $g\rho'$ downwards—*i.e.*, in the gravitational sense ; while the resultant pressure of the surrounding fluid on it will be $g\rho$ upwards. The resultant force downwards will be $g(\rho'-\rho)$, which will be positive, *i.e.*, downwards, if the substituted matter is denser or its specific volume less, and upwards if it is less dense or its specific volume greater than the medium immediately surrounding it.

6. Now take a small mass, say 1 gram, of the medium at r of density ρ, or specific volume v and pressure p, and allow it to change adiabatically till its pressure becomes $p+dp$. Its specific volume becomes $v+\left(\frac{\partial v}{\partial p}\right)_\phi dp$. Where the subscript ϕ implies that the entropy ϕ is kept constant. That is, $v(1-\beta_\phi dp)$ where β_ϕ is the adiabatic compressibility of the

medium at r. In like manner, take an equal mass of the medium at $r+dr$, of density $\rho+d\rho$ or specific volume $v+dv$, and at pressure $p+dp$, and change it adiabatically till its pressure becomes p. Its specific volume becomes

$$v\left(1+\frac{dv}{v}+\beta_\phi dp\right).$$

Now interchange the two equal masses. The forces acting on the two will be opposed to the interchange if the mass brought down has a larger specific volume than v, and the mass carried up a smaller specific volume than $v+dv$. That is, the equilibrium will be stable if

$$v(1-\beta_\phi dp)<v+dv.$$

and

$$v\left(1+\frac{dv}{v}+\beta_\phi dp\right)>v.$$

Both conditions are satisfied if

$$dv+v\beta_\phi dp>0, \qquad (6\cdot1)$$

that is, if

$$\frac{dv}{dr}+v\beta_\phi\frac{dp}{dr}>0,$$

which reduces, since $\frac{dp}{dr}=-\frac{g}{v}$

to

$$\frac{dv}{dr}>g\beta_\phi \qquad (6\cdot2)$$

or the rate of increase of the specific volume of the fluid with distance from the centre at any point must exceed g times the adiabatic compressibility of the medium at that point for stable equilibrium; the two must be equal for neutral and the latter exceed the former for unstable equilibrium.

Since the composition of the fluid has not been specified in the above discussion, the condition for stable equilibrium holds for a fluid mixture of any composition whose specific volume and adiabatic compressibility are known or calculable.

7. As β_ϕ is not the compressibility which is directly measured, it may be calculated from the value of the observed isothermal compressibility β_θ by the help of the relation $\beta_\phi=\frac{\beta_\theta}{\gamma}$, where γ is the ratio of the specific heat at constant pressure to that at constant volume, in cases in which γ is directly determined.

In all cases it can be calculated from the relation

$$\beta_\phi = \beta_\theta - \frac{1}{c_p} \cdot \frac{\theta}{v} \cdot \left(\frac{\partial v}{\partial \theta}\right)_p^2 \quad . \quad . \quad . \quad . \quad (7\cdot1)$$

where c is the specific heat at constant pressure measured in watt units.

For a perfect gas, $\beta_\theta = \frac{1}{p}$, and hence $\beta_\phi = \frac{1}{\gamma p}$; while for liquids $\beta_\phi = \beta_\theta$ to a degree of approximation sufficient for many purposes.

8. If the entropy of a gram of the fluid can be expressed in terms of its specific volume and its pressure, which will be the case if the fluid is homogeneous, we then have

$$\frac{d\varphi}{dr} = \left(\frac{\partial \varphi}{\partial v}\right)_p \frac{dv}{dr} + \left(\frac{\partial \varphi}{\partial p}\right)_v \frac{dp}{dr}.$$

But

$$\frac{dp}{dr} = -\frac{g}{v}.$$

Hence

$$\frac{d\varphi}{dr} = \left(\frac{\partial \varphi}{\partial v}\right)_p \frac{dv}{dr} - \left(\frac{\partial \varphi}{\partial p}\right)_v \cdot \frac{g}{v}. \quad . \quad . \quad . \quad . \quad . \quad . \quad (8\cdot1)$$

Substituting in the condition for stable equilibrium

$$\frac{dv}{dr} > g\beta_\phi,$$

or

$$\frac{dv}{dr} > -\frac{g}{v}\left(\frac{\partial v}{\partial p}\right)_\phi,$$

the value of $\left(\frac{\partial v}{\partial p}\right)_\phi$ in terms of differentials of the entropy

$$\left(\frac{\partial v}{\partial p}\right)_\phi = \frac{\left(\frac{\partial \varphi}{\partial p}\right)_v}{\left(\frac{\partial \varphi}{\partial v}\right)_p},$$

we have stable equilibrium if

$$\frac{dv}{dr} > \frac{g}{v} \cdot \left(\frac{\partial \varphi}{\partial p}\right)_v \Big/ \left(\frac{\partial \varphi}{\partial v}\right)_p,$$

that is, if

$$\left(\frac{\partial \varphi}{\partial v}\right)_p \frac{dv}{dr} > \frac{g}{v}\left(\frac{\partial \varphi}{\partial p}\right)_v \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (8\cdot2)$$

which is the condition that in (8·1)

$$\frac{d\varphi}{dr} \text{ is } +ve. \quad . \quad (8\cdot3)$$

Thus, in a sphere of a homogeneous fluid in stable mechanical equilibrium under its own gravitation, the entropy of unit mass of the fluid must increase as the radius increases. The equilibrium is convective or neutral when the entropy is independent of the radius, and the fluid is unstable if the entropy decreases as the radius increases.

(To be continued.)

XXVI. *The Origin of the Elements.* *By* J. H. VINCENT, *M.A., D.Sc., A.R.C.Sc.* (*Head of the Physics Department, L.C.C. Paddington Technical Institute.*)

RECEIVED FEBRUARY 5, 1920.

THE study of radio-activity has provided details of the origin of a large number of elementary substances. The view that all the elements are genetically connected, the steps occurring as a result of radio-active change, is one that is in the minds of many workers, and has been for some time. In the following Paper evidence is adduced in favour of this view, which is discussed in the light of recent results. The theory is then applied to the problem of representing the atomic weights as functions of Moseley's numbers.

In Figs. 1 and 2 the atomic weights of the 83 elements of the international list* are plotted against Moseley's numbers. These last numbers, in a few cases, are not the exact numbers found by Moseley, but the present accepted numbers, which indicate the total positive charge on the atomic nucleus. In all cases the international atomic weight (0=16) has been plotted, and in most cases the atomic weights of isotopes, either experimentally known from Aston's results,† or assumed to exist to fit in with the view taken in this Paper, are also inserted.

Isotopic Arrays and Classes.

When the atomic weights have been reinvestigated by means of positive-ray analysis, and by the chemical determination of the atomic weights of elements in pure mineral species, it will be possible to draw a diagram in which the atomic weights of the isotopes of each element are plotted as the multiple-valued function of the nuclear charge. The curve which is now obtained by plotting the international atomic weights against Moseley's number is that of a single valued function. This is merely because the atomic weights used are the automatically weighted means of the relative masses of atoms of isotopes.

Thus at present we plot 24·32 as the atomic weight of the element whose atoms have a nuclear charge of 12 electronic units of positive electricity. This is probably the weighted

* "Chem. News," January 23, 1920.

† "Nature," November 27, December 18, 1919, and March 4, 1920; "Phil. Mag.," April and May, 1920.

mean of the isotopes, and instead of a single point with 24·32 as ordinate, we must think of a series of points all with 12 as abscissa, each point representing an isotope of magnesium.

The mean value is replaced by an array of values. This isotopic array may vary in extent over an unknown range.

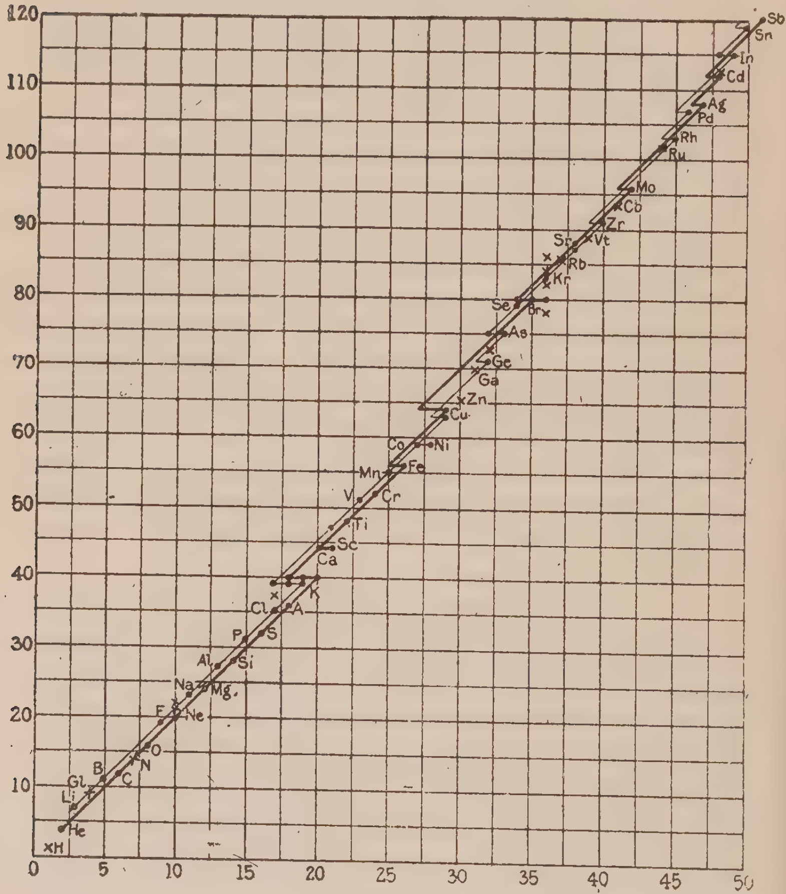


FIG. 1.

In the case of lead it is known to vary over eight units of atomic weight; with chlorine and neon over two units, while krypton has six isotopes varying in relative mass from 78 to 84. What numerical restrictions the separate terms of an isotopic array are subject to is unknown, but with the exception of

hydrogen they all seem to be whole numbers, or to approach very closely to whole numbers.

Aston finds that hydrogen is a "pure" element of atomic weight 1·008 when O=16. Hydrogen is thus in a class apart;

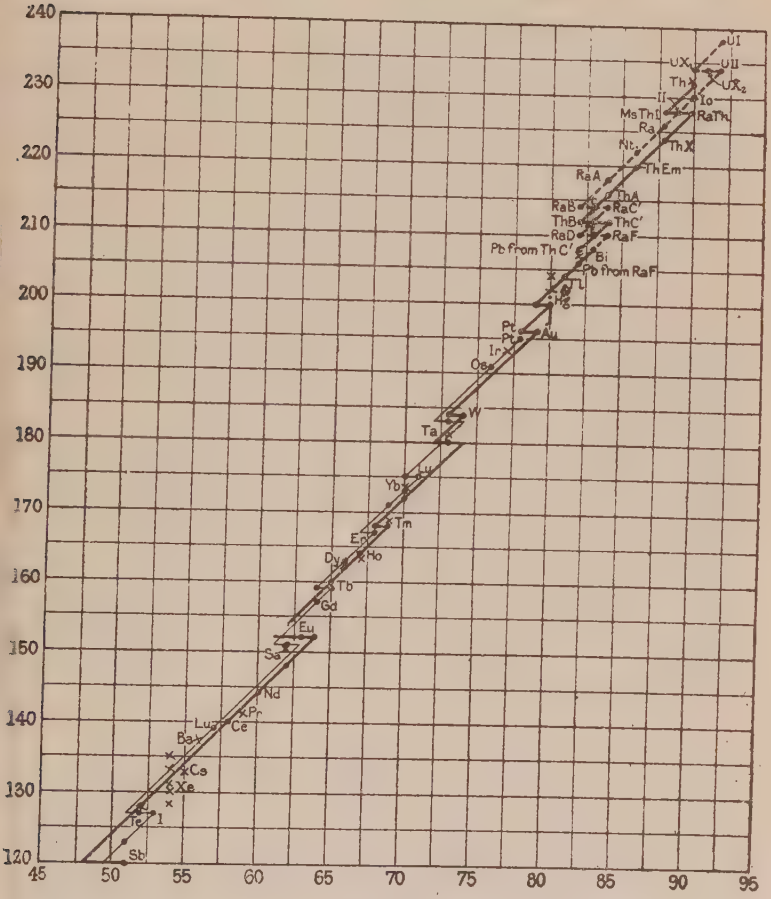


FIG. 2.

other isotopes are grouped into classes indicated by the remainders found by dividing the atomic weights of the isotopes by four.

Prout's hypothesis can then be brought up to date by assuming that the atomic weights of the separate isotopes

(with the exception of hydrogen) are whole numbers when oxygen is taken as 16.

Rutherford has proved that the mass of the alpha particle is very accurately four, and Aston has shown that oxygen is "pure," in the sense that it is not a mixture of isotopes. Thus, without Prout's hypothesis, the assumption that the elements have descended from other elements by the expulsion of alpha and beta particles explains the integral value of the atomic weights of those isotopes whose atomic weights are divisible by four—that is, of class 0 isotopes. The approximation of many atomic weights to whole numbers may be due to the presence of a high percentage of some one isotope. In an isotopic array in which it happens that one of the separate isotopic atomic weights approaches in value to the weighted mean value in virtue of its atoms being present in relatively great numbers, it is convenient to call such an isotope the predominant isotope. Thus, neon 20 is the predominant isotope of the element neon, whose atomic weight is 20·2, this being the weighted mean of the isotopic array.

It should be noted that the points in an isotopic array are not necessarily single points. Isotopes of the same element which have the same atomic weight may differ in descent and in length of life, although chemically indistinguishable and physically inseparable by diffusion.

Numerical investigations which have in the past been ostensibly concerned with the so-called laws of the atomic weights, have in reality been dealing with the relations which hold between the weighted means of isotopic arrays and Moseley's numbers.

Atomic Weight Changes Due to Radio-activity.

In Fig. 2 the derivation of one lead isotope from uranium and of another from thorium have been inserted for illustration; it is sought to show that processes similar to those known to occur in these cases may be going on throughout the whole range of the known elements. In the figures the unit change of atomic nuclear charge is represented on twice the scale of unit change in the atomic weight, so that the loss of an alpha particle is represented by the movement of the indicating point on the diagram downwards along a line at 45 deg. to the axes. For example, UI emits an alpha particle to become UX_1, which is four units lighter than UI, and whose nucleus carries two units less of positive electricity. Changes such as

this may be termed alphatopic, and the same word may be used even when the change in the atomic weight is greater, so long as this is a multiple of four and is accompanied by a movement of two places to the left in the periodic table for every four units loss of atomic weight.

When an atom emits a beta particle its atomic weight is not appreciably affected, but it moves one place to the right in the periodic table. Such a change may be termed a betatopic one, and no confusion will arise if the word is used in a more extended sense. Thus, UX_1 gives out a beta particle to become UX_2, which, on discharging another electron, turns into UII. The motion of the indicating point on Fig. 2, representing the betatopic change from UX_1 to UII is parallel to the axis of Moseley's numbers, and two units long.

Now, the change from UI to UII involves the loss of both kinds of particles; this may be called alphabetatopy. The conditions to be satisfied by an isotope in order for it to be possibly derived by alphabetatopy from another is that its atomic weight must be less than the parent by a number of units divisible by four, and that twice the quotient is greater than the number of places the daughter element has moved to the left of the parent in the periodic table. The use of these three words to indicate the expulsion of helium nuclei, of electrons and of both kinds of particles saves circumlocution. Alternative terms would be heliotopy, electrotopy; but a combination of these two would be very inelegant.

The physical truths outlined above are well known owing to the discoveries and writings of J. J. Thomson, Rutherford, Soddy and their students.

Examples of Similar Relations Among Other Elements.

H. Collins has long since called attention to a number of pairs of elements which are related to each other chemically and by their atomic weights, as if the heavier of the two had produced the other by losing an alpha particle.*

It is well known that most of the international weights which have integral values are either exactly divisible by four or when divided by four have three for remainder. Assuming that in most cases the predominant isotope has the atomic weight of the element, they have been joined in the diagrams by lines indicating possible radio-active changes. In Figs. 1

* "Chem. News," February 28, 1913.

and 2 the thin lines join isotopes whose atomic weights have three as a remainder when divided by four (class 3), and the thick lines join those whose atomic weights are exactly divisible by 4 (class 0).

Description of the Tables and Figures.

Paying attention chiefly to the elements whose atomic weights are given in the international list, Table I. gives a list of the elements in which an isotope of class 0 is present or is assumed to be present, those in which an isotope of class 3 occurs being given in Table II.

Omitting the intensely radio-active bodies, Table I. begins with bismuth.

Before the symbol of each element is given its Moseley's number, and after it is given the accepted atomic weight, while in the third column is given the atomic weight of an isotope of the element.

The fourth column of Table I. contains entries from Table II., and gives the isotopes of elements from that table when the same element occurs in both tables. The last column of Table II. similarly gives isotopes from I.

Alphatopic changes are indicated by a single bar at the left of the tables, betatopic changes by no bar, and alphabetatopic changes by two bars. A branch product is indicated by its inclusion in curved brackets.

Class 0 isotopes are joined by thick zig-zag lines in the figures, the thin lines passing through class 3 isotopes.

It will be seen that the atomic weights of the isotopes are usually either equal to or approach closely to those of the corresponding element. In many cases the atomic weight of the element lies between those of two isotopes of different classes. For example, the atomic weight of antimony is 120·2. The predominant isotope has an atomic weight of 120, the excess of the atomic weight of antimony above this being accounted for by the presence of a class 3 isotope of mass 123. The atomic weight of calcium is accounted for by the presence of two isotopes 40 and 44 of the same class.

Only one isotope (200) of mercury has been included in the tables; another of class 3 of atomic weight 199 alphatopically related to platinum 195 would join up with the list in Table II. Aston has provisionally given the isotopes of mercury as 197-200, 202, 204. Xenon probably has one isotope of class 0 and two of class 3 which are omitted from the tables. The

TABLE I.

Mosley's number.	—	Atomic weight of chemical element.	Atomic weight of class 0 isotope.	Atomic weight of class 3 isotope. *See* Table II. & remarks.
83	Bi	208·0	208	......
81	Tl	204·0	204	*See* also 3 lines lower
79	Au	197·2	200	......
80	Hg	200·6	200	......
78	Pt	195·2	196	195
79	Au	197·2	196	......
74	W	184·0	184	......
73	Ta	181·5	180	183
70	Yb	173·5	172	175
68	Er	167·7	168	167
69	Tm	168·5	168	171
67	Ho	163·5	164	......
65	Tb	159·2	160	159
63	Eu	152·0	152	......
64	Gd	157·3	152	159
62	Sa	150·4	148	151
60	Nd	144·3	144	......
58	Ce	140·25	140	......
52	Te	127·5	128	127
51	Sb	120·2	120	123
48	Cd	112·40	112	115
47	Ag	107·88	108	......
42	Mo	96·0	96	......
38	Sr	87·63	88	87
36	Kr	82·92	84	83
34	Se	79·2	80	79
35	Br	79·92	80	......
(36	Kr	82·92	80	83)
29	Cu	63·57	64	63
26	Fe	55·84	56	......
24	Cr	52·0	52	......
22	Ti	48·1	48	......
20	Ca	40·07	44	*See* also 4 lines lower.
(21	Sc	44·1	44	47)
18	A	39·9	40	......
19	K	39·1	40	39
20	Ca	40·07	40	......
18	A	39·9	36	......
16	S	32·06	32	......
14	Si	28·3	28	......
12	Mg	24·32	24	......
10	Ne	20·2	20	......
8	O	16·00	16	......
6	C	12·005	12	......
2	He	4·00	4	......

TABLE II.

Moseley's number.	—	Atomic weight of chemical element.	Atomic weight of class 3 isotope.	Atomic weight of class 0 isotope. *See* Table I. & remarks.
78	Pt	195·2	195	196
76	Os	190·9	191	......
73	Ta	181·5	183	180
70	YB	173·5	175	172
71	Lu	175·0	175	......
69	Tm	168·5	171	168
68	Er	167·7	167	168
66	Dy	162·5	163	......
64	Gd	157·3	159	152
65	TB	159·2	159	160
62	Sa	150·4	151	148
57	La	139·0	139	......
52	Te	127·5	127	128
53	I	126·92	127	......
51	Sb	120·2	123	120
50	SN	118·7	119	......
48	CD	112·4	115	112
49	In	114·8	115	......
46	PD	106·7	107	......
45	Rh	102·9	103	......
40	Zr	90·6	91	......
38	Sr	87·63	87	88
36	Kr	82·92	83	80
34	Se	79·2	79	80
32	Ge	72·5	75	*See* also 2 lines lower
33	As	74·96	75	......
32	Ge	72·5	71	......
29	Cu	63·57	63	64
27	Co	58·97	59	......
(28	NI	58·68	59)	......
25	MN	54·93	55	......
23	V	51·0	51	......
(21	Sc	44·1	47	44)
17	Cl	35·46	39	*See* also 2 lines lower
19	K	39·1	39	40
17	Cl	35·46	35	......
15	P	31·04	31	......
13	Al	27·1	27	......
11	Na	23·00	23	......
9	F	19·00	19	......
5	B	10·9	11	......
3	LI	6·94	7	......

two isotopes of argon found by Aston have places in Fig. 1 and Table I.

In drawing up the tables alternative methods of connection have not been given. Wherever alphabetatopic changes occur (shown by a double bar at the side of the tables), these alternative paths may occur in the figures, and thus isotopes of the same mass but different pedigree may exist.

All the isotopes of all the elements to be found in Figs. 1 and 2 are not given in the tables, only those which are helpful in explaining the atomic weights of the elements. Thus, in the case of tungsten, only one isotope is mentioned in the tables, this is 184, being of the same value as the atomic weight of the element; the other possible isotopes which can be found by the inspection of Fig. 2 are omitted.

It is a very remarkable circumstance that in the international list of well-determined atomic weights there are so few cases which suggest betatopy. This may be because the substances emitting beta rays are short lived when compared with those emitting alpha rays. The occasional occurrence of a short-lived substance emitting slow alpha rays is suggested by the absence of members from alphatopic series. Thus, the series at the end of Table I. beginning with Ca 40 has no known member corresponding with Moseley's number 4. This is a short-lived isotope of glucinum, which splits up into two helium atoms.

In Figs. 1 and 2 the atomic weights of elements not mentioned in Tables I. and II., which have atomic weights suggesting the presence of predominant isotopes of class 1 (with remainder 1 when divided by 4) and of class 2 (with remainder 2), are shown by crosses; the same sign is used when the atomic weight differs appreciably from that of its isotopes.

Missing Elements.

There are a few Moseley numbers (75, 72, 61, 43) below those of the well-known radio-active elements, isotopes corresponding to which are not known. Their fewness is simply explained on the theory adopted in this Paper. If an element has no long-lived representatives from any of the lines of descent, then it would be so rare as to be difficult to separate and identify by the older methods. That the isotopes from the different families should all happen to be short lived is unlikely, and thus such cases are rare. Even such substances will, however, be tracked down when positive ray analysis is

applied to mineralogy. This method, due to Sir J. J. Thomson, has been recognised by him from the first as an exceedingly powerful instrument of research. It is, however, one requiring elaborate equipment, and is at present only being applied in the Cavendish Laboratory. It is easy to see that very far-reaching investigations are within its power; when the full results have been gathered, figures and tables on the general plan of those accompanying this Paper will be an essential part of the doctrine of matter; but these figures and tables of the future will be drawn up without any taint of mere arithmetical ingenuity. It is quite possible, however, that, even in the light of wider knowledge, the two main lines of descent traced in the present figures will be corroborated, and more especially it is probable that the main alphatopic series of classes 0 and 3 will be experimentally verified.

Other Elements.

Tables I. and II. include the great majority of the elements not recognised as radio-active. The majority of the remaining elements fall into class 1, few having atomic weights indicating predominant isotopes of class 2. The Moseley numbers are, in general, so far apart in these two classes that it would be largely a matter of choice were figures and tables to be drawn up for them. It may be noted that a magnesium of weight 25 would be related alphatopically to chlorine 37, and, together with magnesium (24), would account for the atomic weight of ordinary mixed magnesium 24·32. This would remove the only marked deviation of the value of the atomic weights of the elements in Tables I. and II. from a value either very near to that of an isotope or lying between those of two isotopes.

Although nitrogen has an atomic weight of 14·01, Aston finds that it is a "pure" element, having only one isotope, 14. Rutherford has found that when nitrogen is bombarded with alpha rays hydrogen is probably produced. He suggests that the nitrogen nucleus consists of three helium nuclei, each of atomic mass 4, and either two hydrogen nuclei or one of mass 2.* The remnant left after three helium nuclei are subtracted from the nitrogen nucleus would be, if this loss of alpha particles were the result of radio-activity, a substance of mass 2 isotopic with hydrogen. The result of smashing up the atom by bombardment with alpha particles would

* "Phil. Mag.," June, 1919.

probably be different from that consequent on the action of automatic disintegration.

Are Most Elements Radio-active?

It may be objected that a scheme such as is outlined in this Paper is put out of court at once by the remark that, with a few possible exceptions, no radio-active elements are known whose Moseley numbers are less than 81. This objection is not, however, conclusive. It is to a large extent met by the view that a substance may be radio-active without its radio-activity being detectable by the methods at present used. Rayless changes are recognised in the subject of radio-activity, and are inferred from the behaviour of the parent and the products. If the particles are emitted with a low enough velocity, radio-active changes may proceed undetected. This is an acknowledged principle, and will be again discussed when dealing with a most valuable suggestion of Soddy.

If, however, the rate of transformation is small, even though the few particles are detectable, the investigation of such cases is not likely to attract many experimenters while problems remain unsolved in connection with the more actively disintegrating substances.

As the atomic number is lessened the energy of the emitted particle would decrease, and the products would tend to become longer lived. On the whole, then, we should expect that, as the world aged, the proportion of the lighter atoms would increase, some of the heavier atoms disappearing altogether. The ancestors of thorium and uranium seem already to have died away. These bodies were probably actively radiating, and they have not survived in sufficient quantity to be detected even by their radio-activity. The same fate awaits all the known radio-active bodies. According to the view adopted in this Paper, all other elements are likewise doomed to disappear, with perhaps the exception of the different isotopes of hydrogen and helium. The helium as formed would, according to the theory of Johnstone Stoney, tend to leave the earth owing to its gravitational force being insufficient to retain a gas of such low density. The same causes would operate in the case of the other final products of atomic katabolism, and when most of the present contents of the earth had degenerated to the various isotopes of hydrogen and helium, these would leak away until the loss of temperature by radiation and convection brought this loss to an end.

The great majority of the known radio-active substances are so rare that they could not have been discovered and studied apart from their radio-activity. As we deal with elements of less atomic number, many of these are sufficiently long lived to enable them to be investigated by ordinary chemical methods, their length of life being correlated with their feeble radio-activity. Some of the lighter elements have been regarded as radio-active by many physicists ; rubidium and potassium may be mentioned as instances. According to Fig. 1, ordinary potassium (39·10) is a mixture of isotopes of mass 39 and 40. This latter is short lived compared with the predominant isotope, and emits only beta rays. This view is not antagonistic to experimental results.

Several minerals are known which contain notable quantities of helium, but which exhibit no radio-activity. From the standpoint now being taken, these minerals are regarded as containing one or more substances undergoing atomic disintegration, the products of the changes being given off with insufficient energy to ionise gases or to produce scintillations.

Some Radio-active Processes May be Reversible.

Although the above is apparently the future history of the earth and its contents so far as can be deduced from what is known of radio-activity at present, and from the assumption that radio-activity is an almost universal property of the elements, it also assumes that radio-active processes are irreversible, and that there are no other processes which might be regarded as atomic anabolism.

The question of the possible reversibility of radio-active processes is one of great interest. Soddy has suggested* that, in view of the fact that the quantity of radium in the common rocks of the earth's crust is so great that a layer of these rocks only a few miles thick would supply the heat lost by the earth, a process of atomic upbuilding, with the necessarily enormous absorption of energy, is taking place in the interior of the earth. The assumption that the common elements are nearly all continually disintegrating increases the necessity for some theory to account for the heat thus generated. This may be met in different ways, but the most satisfactory method of meeting the difficulty is to imagine that the atomic upbuilding which Soddy suggests is sufficiently active to absorb the

* B. A. Report, 1906, p. 130.

energy due to the disintegration of ordinary matter as well as that due to the more actively radiating substances.

In a lecture before the Chemical Society, December 19, 1918,* Soddy mentioned irreversibility as one of the chief features of radio-active change, so that the processes going on in the interior of the earth do not in Soddy's opinion include reversed radio-activity, some other type of atomic upbuilding being probably implied.

From Rutherford's experiments with nitrogen, the effect of subjecting it to alpha rays seems to be that some of the nitrogen atoms are broken up and hydrogen produced. No experiment, so far as I know, has shown that the alpha particle sticks to the atomic target to produce a reversed alphatopic change. That a reversed betatopic change may be possible is shown by an investigation by C. G. Darwin.† His Paper is concerned with the orbit of a beta particle as it passes a Rutherford nucleus, and he concludes that in certain cases it becomes a spiral going right into the centre. "Numerical calculation shows that these cases should be of fairly frequent occurrence." Now, what effect will an event like this have on the atom, the nucleus of which has absorbed a beta particle? It appears not at all unlikely that the atom would keep its atomic weight sensibly unchanged, and all its chemical properties would alter.

Its new behaviour would be that of the place of one to the left of that previously occupied in the periodic table; such would be a reversed betatopic change. For instance, lithium would by such a single reversed betatopic change become an isotope of helium. It would be a monatomic inert gas having a similar visible spectrum to helium, although the wavelengths would not be absolutely identical. Similarly sodium would become isotopic with neon, gold could be turned into an isotope of platinum, and if the process were capable of repetition, lead into gold. It may be that the results of experiments by Ramsey, Collie and Patterson,‡ in which neon was found to be produced in vacuum tubes, and also the somewhat similar results of other investigations, may be explained as being examples of betatopy.

The result of adding negative electrons to the nucleus of an atom which has a low Moseley number might conceivably

* "Chem. News," February 21, 28, and March 7, 1919.
† "Phil. Mag.," February, 1913.
‡ "Chem. News," February 14, 1913.

be the disappearance of the substance altogether, Suppose the nucleus of a helium atom consists merely of two positive electronic charges, and owes all its inertia to the concentration of these charges, then, if it received two negative electrons, these might cause it to vanish, an equivalent amount of energy being radiated into space. With hydrogen on the same view a single addition of a negative electron might cause annihilation.

Difficulty with Respect to the Separate Classes.

Another possible objection may be that no general scheme of the origin of the elements has been attempted.

The answer to this is that such a scheme would be premature. My main object is far less ambitious, in that I attempt to support the view that, granted the possibility of the unrecog nised emission of alpha and beta particles, this will account for the existence of the great majority of the dominant isotopes of the elements. I freely admit that there are very grave difficulties in going much more into detail at present.

One trouble is that the various hypothetical radio-active families corresponding to the classes of isotopes cannot be linked up together by alphatopy and betatopy alone, although these are at present the only known methods of atomic change.

This difficulty, with perhaps its solution, is referred to by Soddy in his Chemical Society lecture. I give his own words : " A heavy atom like oxygen, for example, if expelled as a radiant particle, might not attain sufficient velocity to ionise gases, or, even if it did, the range over which the ionisation would extend, as we know from the ionisation produced by the recoil atoms, would be extremely small." After commenting on the fact that hydrogen is never emitted, he goes on : " It has always seemed to me a possibility that some genetic connection may exist after all between thorium and uranium, although I have never been able to frame even a possible mode of so connecting these two elements. With a difference of atomic weight of six units, it is impossible to pass from one to the other by addition or expulsion of helium atoms alone." Now it may be that in the above quotations we have first a solution and then an example of the type of problem to which it is the key. If the whole of the elements are to be linked up together in a connected series of radio-active families, then it is essential to assume that other than alpha and beta particles are emitted. If we reject the hydrogen nucleus,

because if it were ejected it would probably have been already discovered, then we must assume the existence of at least one radiant particle whose weight is an odd number. But for a similar reason to that for the rejection of hydrogen we must also reject other nuclei of low atomic weight. This forces us to assume provisionally that there is at least one other type of emitted particle of odd atomic weight greater than 4, the justification for this assumption being that such particles may be discharged from members even of actively radiating series without having been detected.

Applications of the Theory.

One of the tests of the usefulness of a theory is that it should suggest further experiments. One such experiment is a search for the reproduction of helium in the contents of minerals freed from helium.

Lord Rayleigh found* that beryl, although inactive, contained a very considerable quantity of helium, and he suggested that this was a case of rayless change. It might be possible in this or other suitably selected cases to detect the gradual reformation of helium after freeing the materials from all detectable traces of the gas.

Another consequence of the theory is that, since atomic katabolism involves the liberation of energy, the steady temperature attained by a body in an enclosure whose walls are at constant temperature would, in general, differ from that of the enclosure, while thermally isolated bodies would rise in temperature. It might be that the slowness of the emitted particles is not always accompanied by a low rate of disintegration, and thus temperature effects might disclose the progress of disintegration.

It such cases it is not impossible that slowness of change might be to a large extent compensated for by the fact that the experiments would not be limited to the use of small quantities of matter. I am unaware of any accurate tests of the assumption frequently made that when bodies of different materials reach a steady state in an enclosure they are all at the same temperature.

The Atomic Weight as a Function of Moseley's Number.

It may fairly be asked of a theory that it should clear up difficulties which have attracted previous attention.

* R. G. Strutt, "Nature," February 21, 1907.

Now, there is a considerable body of literature which has been concerned with the remarkable approximation to regularity in the atomic weights of the elements of low atomic weight. I think that the large majority of the very numerous and interesting connections between the chemical properties and atomic weights of these elements, and also the very close way in which the atomic weights follow simple arithmetical rules, is immediately explicable on the theory advocated. All these connections may be summed up and explained by the view that nearly all the elements of low-atomic weight have predominant isotopes which belong to well-marked alphatopic series of classes 0 and 3.

Rydberg's formula is one of the best known. Comstock* gives a comparison between the formula and the atomic weights for 1907.

If M is the number in the system, the atomic weight is, according to Rydberg's formula, $2M$ and $2M+1$ alternately, beginning with helium, for which M is 2. From helium to scandium, Rydberg's M is Moseley's number, so that $2M$ gives the successive members of class 0 isotopes and $2M+1$ the successive members of class 3 isotopes.

It is possible to see a reason why a long alphatopic series uninterrupted by betatopy should start from calcium 40. This isotope consists of 10 helium nuclei held together in some unknown way. There are no beta particles present either in calcium or in its successive products, except any that may be closely united with the constituent helium nuclei. Betatopy cannot thus occur. On this view, the number of detachable beta particles is $W/2-n$ when the class of an isotope is even (0 or 2), and is $(W+1)/2-n$ when the class is odd (1 or 3).

Another matter which the present view clears up is, why the curve obtained by plotting atomic weights against the order of magnitude in a list of atomic weights should be a somewhat regular curve convex to the axis on which the order is marked.†

Moseley's numbers are nearly all occupied, and as the order of magnitude almost invariably follows the chemical order (which is that also of Moseley), the curve is not much altered if the abscissæ are Moseley's numbers. Now, the ordinates on this curve are the weighted means of the atomic weights of each isotopic array. Any indicating point on Figs. 1 and

* "Phil. Mag.," January, 1908.

† Vincent "On a General Numerical Connection between the Atomic Weights." "Phil. Mag.," July, 1902.

2, so far as is known at present, can move in only two ways. It can move two units to the left or four down; it can move one place to the right.

These two steps can be taken in any order or number when Moseley's number is not small. The mean position of the points of the array must at first descend by paths steeper than that due to alphatopy alone.

The tendency to discharge beta particles becomes less as the number of these decreases, so that the curve gets less steep as the mean atomic weight decreases. This will go on until betatopy becomes no longer possible, as in the case of the nucleus of calcium (40), when the remainder of the descent will be accomplished by the expulsion of alpha particles. The smoothed curve of the path of descent of an isotope of one class will thus be steep when the mass is great; it will get less steep owing to the decrease in the number of beta particles, and will join the straight line given by the final alphatopic series.

The smoothed curve of the mean atomic weight regarded as a function of Moseley's number will have the same general character.

Minet* noted that the curve obtained by plotting the atomic weights as a function of the order of magnitude consisted of two parts, a small first part comprising the elements from helium to calcium, for which $y=1{\cdot}985x$, and the second part given by $y=x^{1{\cdot}23}$. The general explanation of relations such as the above is clear. The straight line is a consequence of alphatopy, the curved one of alphabetatopy; the point where they meet indicates the exhaustion of the supply of beta particles.

In Fig. 3 the atomic weights are plotted against Moseley's numbers. On the scale used the line $y=2x$ is at 45 deg. to the axis, and is shown on the diagram.

Aston's provisional values for isotopes are indicated by crosses, those he regards as certain by circles, while the accepted atomic weights are given by dots.

The straight line is that upon which class 0 isotopes fall when their supply of beta particles is exhausted. A parallel straight line drawn one unit higher is that upon which the class 3 isotopes fall when they enter upon their final alphatopic series of changes.

* "C. R.," 144, 8, 1907.

The curved line in Fig. 3 is the graph of

$$W_n = pn^q$$

where W_n is the atomic weight and n is Moseley's number; p and q are constants having the values 1·36 and 1·14 respectively.

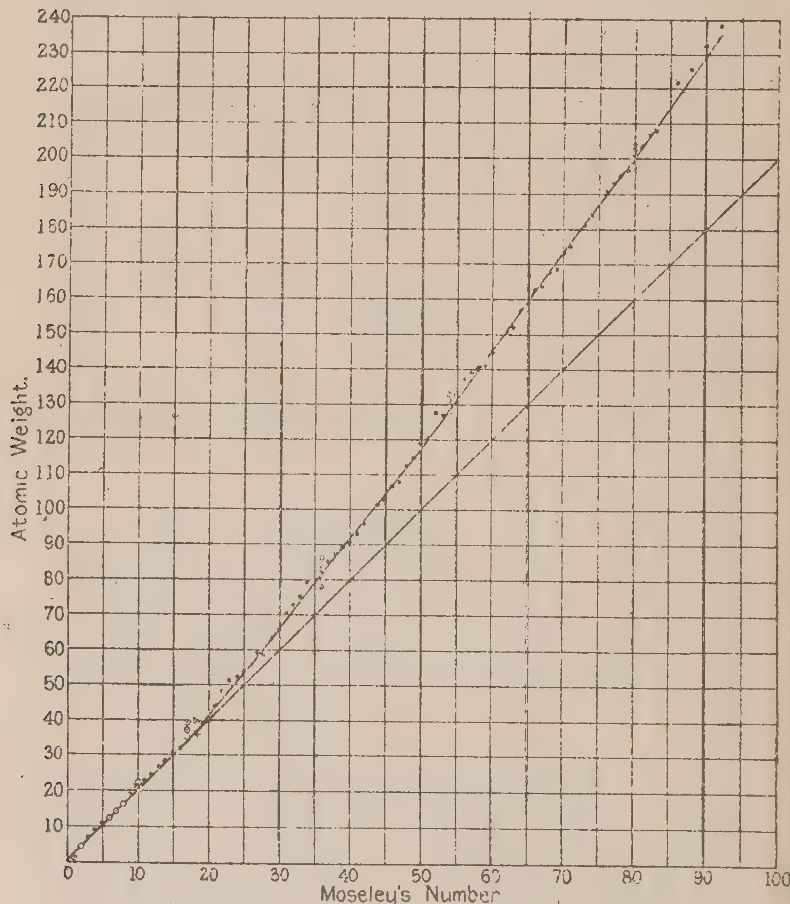


Fig. 3.

This curve cuts the straight line

$$W_n = 2n$$

near the value $n=16$. The portion of the curve below the straight line is not shown in the diagram.

Summary.

The atomic weights are regarded as the weighted mean values of the atomic weights of the isotopes of the elements; but it is assumed that, as a rule, the atomic weight is near that of some one isotope. Figures and tables are drawn up showing how this accounts for the values of a large number of atomic weights, if one also assumes that the weights and positions in the periodic table of any isotope are conditioned by laws similar to those holding in the recognised radio-active families. The elements are all supposed to be derived from parent elements by processes known to occur in actively radiating families; but their radio-activity is not in general detectable by the usual means owing to the velocity of expulsion of the particles being low.

The possibility of the reversibility of some radio-active processes is regarded favourably. The various difficulties in connection with the views advocated are discussed, and some suggestions for experiments are made. Finally, the theory is used to explain the so-called laws of the atomic weights of elements of low atomic weight, and the shape of the curve obtained when the atomic weights are plotted against Moseley's numbers.

Note added June 15, 1920.

Since this Paper was sent to the Society, Aston has published a letter in "Nature" (March 4, 1920) and two Papers ("Phil. Mag.," April and May, 1920). In revising the manuscript I have kept Aston's results in view.

DISCUSSION.

Prof. A. O. Rankine (partly communicated): During the last few months discovery in physics has been making very rapid strides, particularly in relation to the dimensions and constitution of atoms. There are, in addition to the remarkably copious information regarding isotopes obtained by Dr. Aston, two other equally fruitful researches, the results of which have been recently published. I refer to the connection between atomic dimensions and atomic number revealed last month by Prof. W. L. Bragg at the Royal Institution, and to Sir E. Rutherford's discovery—announced in the Bakerian lecture—of a new isotope of helium obtained by disintegrating oxygen. In such circumstances, Dr. Vincent may be regarded as rather bold in having presented his Paper when he did. But I have read it with very great interest, and feel convinced that, although the views expressed will have to be modified in details as discovery proceeds, these new discoveries are themselves most likely to point to connections between the elements on a broad basis similar to the lines indicated by Dr. Vincent. It may, indeed, turn out that the changes suggested by the Author are not actually spontaneous or radio-active, but that they may be effected by the supply of energy from outside. There is just one criticism of Dr. Vincent's procedure which I feel bound to make. I do not see what justification there

is in assuming that every element which is a mixture of isotopes is bound to have a predominant isotope which has an atomic weight equal to the integer nearest to the mean atomic weight of the mixture. There is, indeed, definite evidence that this is not always the case. I am not sure whether Dr. Aston actually refers to it in his recently published Papers, but he has told me, in reply to a question on this very point, that in the case of bromine, the accepted atomic weight of which is 79·92, there is no isotope of weight 80, but that bromine is a mixture in very nearly equal proportions of two isotopes 79 and 81. Thus there is no bromine of zero isotopy, as assumed by Dr. Vincent. In conclusion, I would like to ask the Author whether the new isotope of helium—*i.e.*, an atom of mass 3 and charge 2—which Sir E. Rutherford has succeeded in ejecting from oxygen by alpha ray bombardment, will be found useful in explaining the end products of the radio-active changes contemplated, particularly those specified as class 3 isotopes.

Sir W. Bragg asked if any relationship similar to those for classes 0 and 3 held among elements of classes 1 and 2. He observed that, where lines of descent were given, the differences in atomic weight were nearly 4 or multiples of 4—*e.g.*, rhodium and palladium. This appeared to bear out the theory.

Dr. Vincent, replying to Prof. Rankine, said it was purely an assumption on his part to take the predominant isotope as always existing. It was very rarely indeed that it did not fit in with the facts. He had purposely kept off the subject of end products in the meantime. In reply to the President, there were some well-marked alphatopic series among classes 1 and 2.

XXVII. *An Electrical Hot-Wire Inclinometer.* *By* J. S. G. THOMAS, *M.Sc.* (*Lond.*), *B.Sc.* (*Wales*), *A.R.C.S.*, *A.I.C.*

(COMMUNICATED BY D. OWEN, D.Sc.)

RECEIVED MARCH 30, 1920.

IN a recent communication to the Society,* the author described a type of directional hot-wire anemometer, consisting essentially of two equal, fine, platinum wires arranged parallel, one behind the other, transversely to the direction of flow of the fluid whose velocity was to be measured. The wires constituted two arms of a Wheatstone bridge, in which a constant current of from 1 to 1·5 ampère was maintained, and it was shown that the galvanometer deflection was reversed on reversing the direction of flow of the fluid past the fine platinum wires. Attention was directed in the Paper to certain heating and cooling effects experienced by the wires due to the passage of the stream of fluid, and to the free convection currents rising from the wires. This subject of the free convection current is treated in considerable detail in a further Paper by the author.† The present Paper treats of certain characteristics of a type of hot-wire inclinometer closely resembling the directional hot-wire anemometer referred to above, the free convection currents from the wires producing the cooling and heating effects to which the respective wires are subjected.

Experimental.

For the purpose of the present work, a hot-wire inclinometer was constructed as shown diagrammatically in Fig. 1. A closed chamber, *CE*, was formed of a cylindrical brass tube about 2 cm. in diameter and about 5 cm. long, closed at its ends by the screw caps *C* and *E*. Two platinum wires, *A* and *B*, about 0·1 mm. diameter, were inserted into the chamber in the manner shown, about 1 mm. apart. The wires were as nearly as possible equal in length to the diameter of the tube, and were electrically aged by passage through them of a current of about 1·5 amperes for two hours. They were attached to copper rods by means of the minimum quantity of silver solder affording smooth and secure junctions. The method of inserting the copper rods in the ebonite plugs *Q* and *R* is shown in the figure. Further

* Proc. Phys. Soc., XXXII., pp. 196-207, 1920.
† "Phil. Mag.," Vol. XXXIX., No. 233, pp. 505-534, 1920.

constructional details may be gathered from one of the Papers referred to above.* In the present connection, it was important to affix the wires A and B to the respective pairs of copper rods so that the wires were initially coplanar, and remained in the original diametral longitudinal section of the chamber on passing a current from 1 to 1·5 ampères through them. This was readily effected by suitably turning the appropriate copper supporting rod. For this purpose the cap C was removed, so that the bending of the wires owing to their expansion due to heating by the current could be observed, and increased

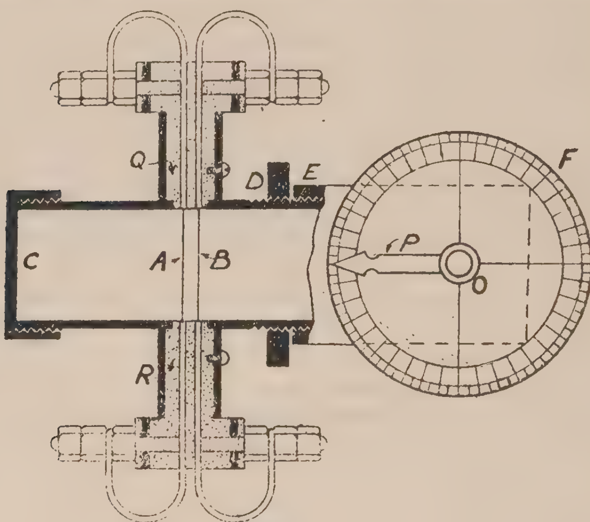


Fig. 1.

accuracy of the setting of the wires could be secured by viewing the wires with a low power microscope. Departure of the wires from the desirable coplanar condition may be entirely prevented by mounting the wires in a spring support, so that they are kept throughout in a taut condition when an electric current is passed through them. For measurements of the greatest precision, this refinement is probably necessary, although experience with the type of mounting shown in Fig. 1 showed that if precautions as above were taken to see that the wires, initially coplanar, remained coplanar on expansion, the indications of the instrument, when employed as explained later, were remarkably constant. The wires after

* "Phil Mag.," 1920, *loc. cit.*

repeated heatings and coolings were found to expand and contract, so that the plane defined by them was throughout the same. The magnitude of their expansion was, of course, conditioned by the heating current employed, and could be, if desired, reduced by employing a smaller heating current. The chamber CE was carried by an arm capable of being revolved round a horizontal axis O. A pointer, P, moving over a divided circle of 12 in. diameter enabled the inclination of the arm to the horizontal to be determined. The divided circle was marked at 1 deg. intervals. CE was secured in position on this arm by means of the lock nut D, and could be affixed to the arm so that the plane of the wires AB could be inclined at any desired angle to the axis of rotation O. In the present experiments, the plane of the wires was in one series parallel to, and in the other at right angles, to the axis of rotation. Fig. 1 shows the case in which the plane of the wires was at right angles to the axis of rotation. The other case was obtained by rotating the chamber through 90 deg. about its own axis. The wires A and B constituted two arms of a Wheatstone bridge, the remaining arms of which were formed of a resistance of 1,000 ohms connected to one end of B, and a resistance capable of adjustment connected to one end of A. The various resistances were connected in the usual manner. The battery terminals were connected to the appropriate ends of the platinum wires, and the bridge current maintained constant by means of a rheostat inserted in the battery circuit, and could be reversed when desired. A Weston voltmeter of resistance 1,000 ohms could be inserted in parallel with A, so that the resistance of A could be calculated from the known value of the current, and the drop of potential across A. The resistance of B was calculated therefrom in the usual manner, the bridge being balanced.

Theory (a).—Axis of Rotation Horizontal in Plane of Wires and Parallel to the Wires.

Consider, first, the case shown in Fig. 2, in which the plane of the wires is parallel to the axis of rotation O. When O, B and A are in the same horizontal plane as represented in the figure by O, B_0, A_0, the wires are heated by the electric current and cooled by the free convection currents rising from them. The adjustable arm of the bridge is adjusted until balance of the bridge with the wires in the position B_0, A_0 results, the currents being adjusted by means of a rheostat to any desired

value. If, now, the arm is rotated through 90 deg., so that the wires take up the position B_{90}, A_{90}, it is clear that if the current is maintained at its previous value the balance of the bridge will be upset, owing to the fact that the heating effect experienced by A, due to the hot ascending free convection current from B is greater in the position represented by $OB_{90}A_{90}$ than is the case in the horizontal position of the arms represented by OB_0A_0. This increased heating of A in the position $OB_{90}A_{90}$ necessitates a greater heat radiation from A to B and an increased free convection current from B to A.

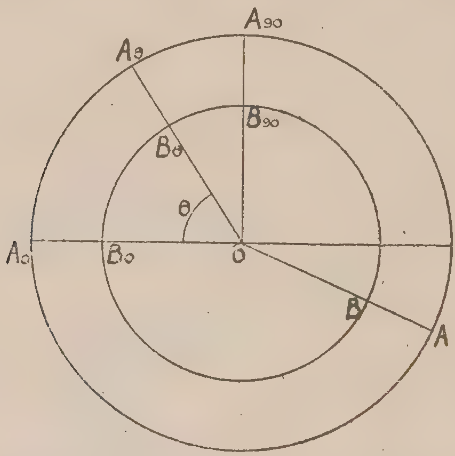


Fig. 2.

In upper semi-circle, Fig. 2, A is the hotter; in lower semi-circle, B is the hotter. There is symmetry of the values of the respective resistances about the vertical diameter through O.

The nett result upon the temperature of B is determined by the relative magnitudes of the free convection cooling effect, and the heating effect due to radiation. It is clear that, on rotation of the arm from the position OB_0A_0 to that represented by $OB_{90}A_{90}$, A comes increasingly within the heating effect of the free convection current from B. B likewise experiences an increased cooling effect due to convection, and the balance of the bridge is correspondingly upset. With passage of the arm through the position represented by $OB_{90}A_{90}$, a condition of affairs which may be best described as the mirrored image in $OB_{90}A_{90}$ of those to the left of this line is passed through. The arm rotating from 180° to 360°, a condi-

tion of affairs in which B is subjected to the heating effect of the free convection current from A is attained. It is clear, therefore, that while the condition of heating or cooling of B and A will be symmetrical about the vertical line through the 90°-270° position of the rotating arm, a symmetrical condition of affairs as concerns the resistances of the respective wires is not to be anticipated about the 0°-180° line.

(b) Plane of Wires at Right Angles to the Horizontal Axis of Rotation of Arm.

The initial positions of the wires are represented in Fig. 3 by

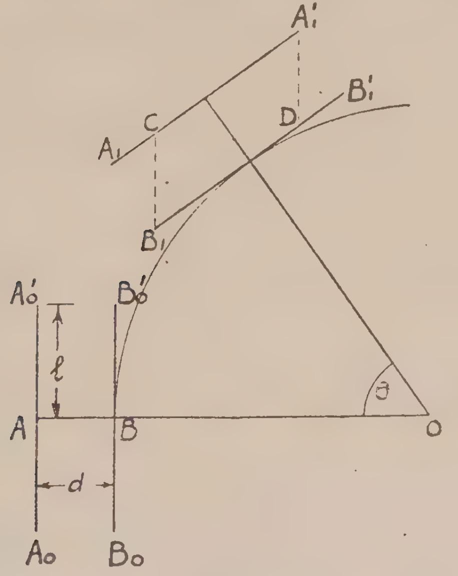


FIG. 3.

B_0B_0' and A_0A_0', the wires being initially vertical. In this position it is clear that if the wires are identical in all respects, each is subjected to the same heating effect due to the free convection current due to the electric current in the other wire. When the arm carrying the wires is rotated through an angle θ, the wires take up the positions indicated by A_1A_1', B_1B_1'. Omitting for the present all reference to alteration in the heating or cooling of either wire due to its own free convection current, in the position A_1A_1' of the wire, $A_1'C$ is immediately above the portion B_1D of the wire B_1B_1', and is

subjected to the heating effect of the free convection current arising therefrom. The portion A_1C of the upper wire is not subject to the full effect of the free convection current arising from the lower wire. The portion A_1C is, therefore, less heated than the remainder of the wire A_1A_1', but the high thermal conductivity of the wire tends to equalise the temperature throughout the wire. The portion A_1C does, however, receive some little heat from the free convection currents arising from B_1B_1', as this free convection current is not confined to the vertical plane extending over B_1B_1'. The spatial distribution of the free convection currents from the fine platinum wires as regards temperature and velocity is being investigated at present. It is evident from Fig. 3 that the free convection current from the portion DB_1' of the wire B_1B_1' will produce very little heating effect in the wire A_1A_1', and it is readily seen that the increased heating effect to which A_1A_1' is subjected on rotation of the arm through the angle θ from the horizontal is proportional to $\alpha \, . \, A_1A_1' - \beta \, . \, A_1C - \gamma DB_1'$, where α is the heat absorbed by unit length of A_1A_1', when vertically above B_1B_1', and β and γ are factors whose significance is clear from the remarks made above. To a first approximation we may assume that $\alpha = \beta = \gamma$, and then the increased heating effect to which A_1A_1' is subjected on rotation of the arm through an angle, θ, from its initially horizontal position is proportional to $2(l - d \cot \theta)$, where $2l$ is the length of each wire and d their distance apart. The increased temperature to which the upper wire is raised thereby is accompanied by (1) an increased radiation towards the lower wire, tending to raise the temperature of the latter, and (2) increased convection currents from the upper and lower wires tending to lower their respective temperatures. The final equilibrium conditions of the wires are determined by the balancing of such opposing tendencies operative on both wires, and, owing to the small mass of the wires, is attained practically immediately the alteration of the inclination of the arm is effected. The theory developed contemplates that any alteration of inclination of the arm is accompanied by a variation in the length of the upper wire immediately above the lower wire. Owing to the finite distance apart of the wires and their finite length, this condition is not secured until the arm has been rotated through an angle $\tan^{-1}\frac{d}{2l}$. This is, of course, decreased by increasing the length of the wires employed, and decreasing

their distance apart. In the present experiments, where $d=0{\cdot}01$ cm. and $l=2{\cdot}05$ cm., the necessary angle of rotation $=\tan^{-1}0{\cdot}0025=1°\,6'$. The remarks already made with regard to the symmetry of the values of their respective resistances about the vertical diameter through O apply equally in the present case as in the former case, and similarly as regards the lack of symmetry about the horizontal diameter.

Results and Discussion.

Tables I. and II. contain the results obtained when the pair of wires was rotated in a vertical plane about a horizontal axis. Table I. contains the results obtained for the case in which the horizontal axis of rotation was in the plane of the wires and parallel to the wires (*see* Fig. 2).

Diameter of cylindrical chamber	2·05 cms.
Length of cylindrical chamber	5·00 cms.
Length of wires employed..........................	2·05 cms.
Diameter of wires employed	0·01 cm.
Distance between wires	0·10 cm.
Ratio arm in bridge	1000 ohms.
Current employed	1·000 amp.
Galvanometer shunt (to reduce sensitiveness)...	11 ohms
Rotating arm in zero position	horizontal.

A, wire at greater distance from axis of rotation.
B, wire nearer to axis of rotation.
Rotation clockwise wires initially as represented by A_0B_0.

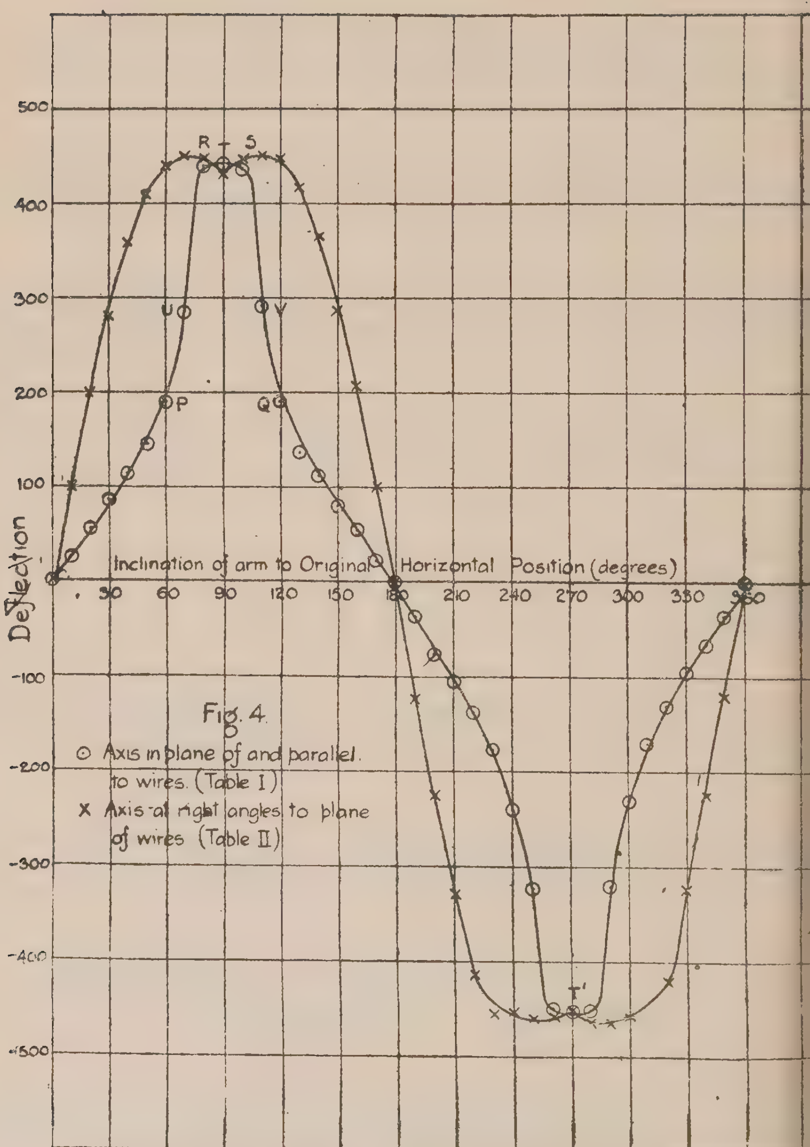
Table II. contains the results obtained when the horizontal axis of rotation was at right angles to the plane of the wires. In the zero position of the rotating arm the wires were vertical (*see* A_0A_0', B_0B_0'; Fig. 3).

The results contained in Tables I. and II. are shown graphically in Figs. 4 and 5. In Fig. 4 the respective deflections are plotted as ordinates against the angles of rotation of the arm from the horizontal position as abscissæ. In Fig. 5 the resistances of the wires are similarly plotted against the angular rotation of the arm. From Fig. 4 it is seen that, in the case where the axis of rotation is parallel to and in the plane of the wires, the sensitivity of the inclinometer is considerably increased when the angle of inclination of the arm to the horizontal attains the value corresponding to the point P. This is due to the fact that, at the inclination represented by the point P, the wire A experiences a greatly increased heating effect, due to the free convection current from the lower wire B. The deflection is very approximately constant at the inclinations corresponding to the points included between R and S.

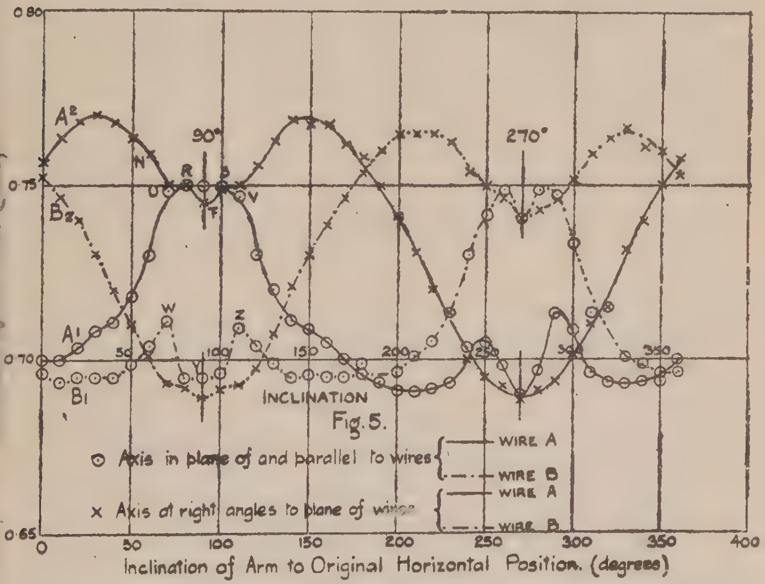
TABLE 1.—*Horizontal Axis of Rotation in Plane of Wires and Parallel to the Wires.*

Inclination of arm to horizontal. (Degrees.)	Deflection.			Balance resistance (ohms).		Drop of potential across outer wire *A* (volt).	Resistance (ohm).	
	To left.	To right.	Total.	Arm horizontal.	Arm inclined.		Wire *A*.	Wire *B*.
0	0	0	0	1004	1004	0·700	0·700	0·697
10	13	14	27	...	1009	0·700	0·700	0·694
20	27	28	55	...	1013	0·704	0·704	0·695
30	42	42	84	1004	1019	0·708	0·708	0·695
40	56	56	112	...	1023	0·711	0·711	0·695
50	73	73	146	...	1029	0·719	0·719	0·699
60	95	95	190	1004	1037	0·730	0·730	0·704
70	144	144	288	...	1053	0·749	0·749	0·711
80	221	219	440	...	1080	0·750	0·750	0·695
90	221	219	440	1004	1080	0·750	0·750	0·695
100	220	216	436	...	1078	0·750	0·750	0·696
110	146	145	291	...	1054	0·747	0·747	0·709
120	95	95	190	1004	1037	0·730	0·730	0·704
130	69	69	138	...	1030	0·720	0·720	0·699
140	56	55	111	...	1023	0·711	0·711	0·695
150	39	40	79	1004	1019	0·709	0·709	0·696
160	28	28	56	...	1014	0·705	0·705	0·695
170	12	12	24	...	1008	0·700	0·700	0·695
180	0	0	0	1004	1004	0·699	0·699	0·696
190	− 18	− 18	− 36	...	999	0·693	0·693	0·694
200	− 36	− 36	− 72	...	991	0·691	0·691	0·697
210	− 53	− 53	−106	1004	986	0·691	0·691	0·701
220	− 68	− 68	−136	...	981	0·692	0·692	0·705
230	− 88	− 88	−176	...	974	0·694	0·694	0·713
240	−119	−119	−238	1004	964	0·704	0·704	0·730
250	−161	−160	−321	...	951	0·705	0·705	0·741
260	−224	−225	−449	...	932	0·698	0·698	0·749
270	−227	−225	−452	1004	931	0·690	0·690	0·741
280	−225	−225	−450	...	931	0·697	0·697	0·749
290	−158	−159	−317	...	953	0·713	0·713	0·748
300	−116	−114	−230	1004	966	0·708	0·708	0·733
310	− 85	− 84	−169	...	976	0·696	0·696	0·713
320	− 65	− 65	−130	...	981	0·694	0·694	0·715
330	− 48	− 48	− 96	1004	988	0·693	0·693	0·701

of arm to horizontal. (Degrees.)	To left.	To right.	Total.	Arm horizontal.	Arm inclined.	potential across outer wire A (volt).	Wire A.	Wire B.
0	0°	0	0	1006	1006	0·757	0·757	0·753
10	50	50	100	...	1022	0·764	0·764	0·747
20	101	98	199	...	1038	0·768	0·768	0·740
30	141	140	281	1006	1053	0·770	0·770	0·731
40	180	178	358	...	1065	0·768	0·768	0·721
50	205	203	408	...	1075	0·763	0·763	0·710
60	220	220	440	1006	1081	0·759	0·759	0·702
70	225	225	450	...	1083	0·750	0·750	0·693
80	224	223	447	...	1034	0·750	0·750	0·692
90	215	215	430	1006	1081	0·745	0·745	0·689
100	222	221	443	...	1083	0·749	0·749	0·692
110	225	224	449	...	1083	0·750	0·750	0·693
120	225	223	448	1006	1083	0·756	0·756	0·698
130	209	210	419	...	1077	0·763	0·763	0·708
140	182	181	363	...	1067	0·769	0·769	0·721
150	143	146	289	1006	1053	0·768	0·768	0·730
160	104	103	207	...	1040	0·768	0·768	0·739
170	50	51	101	...	1022	0·762	0·762	0·746
180	0	0	0	1006	1006	0·758	0·758	0·754
190	−61	−60	−121	...	987	0·750	0·750	0·760
200	−113	−113	−226	...	970	0·742	0·742	0·765
210	−164	−164	−328	1006	955	0·731	0·731	0·765
220	−207	−205	−412	...	941	0·720	0·720	0·765
230	−228	−226	−454	...	935	0·713	0·713	0·763
240	−227	−226	−453	1006	929	0·700	0·700	0·754
250	−230	−230	−460	...	927	0·695	0·695	0·750
260	−228	−229	−457	...	928	0·693	0·693	0·747
270	−226	−225	−451	1006	930	0·689	0·689	0·741
280	−232	−232	−464	...	931	0·692	0·692	0·743
290	−234	−233	−467	...	930	0·694	0·694	0·746
300	−229	−230	−459	1006	933	0·702	0·702	0·752
310	−222	−222	−444	...	936	0·710	0·710	0·759
320	−211	−209	−420	...	940	0·715	0·715	0·761
330	−162	−161	−323	1006	955	0·732	0·732	0·767
340	−112	−111	−223	...	971	0·740	0·740	0·762
350	−60	−60	−120	...	987	0·750	0·750	0·760
360	0	0	0	1006	1006	0·758	0·758	0·753


Deflection
Inclination of arm to Original Horizontal Position (degrees)
Fig. 4
⊙ Axis in plane of and parallel to wires. (Table I)
X Axis at right angles to plane of wires (Table II)
500
400
300
200
100
-100
-200
-300
-400
-500
30
60
90
120
150
180
210
240
270
300
330
360
R
S
U
V
P
Q
T'

It will be seen from Fig. 5 that the resistance, and consequently the temperature, of the wire A is very approximately constant over the region represented by the points U, R, S, V. The greater deflection corresponding to R compared with that represented by U arises, as seen from Fig. 5, from the greater cooling which the lower wire B experiences when the upper wire A is in the position corresponding to R compared with that corresponding to U. This is clearly seen in Fig. 5 from the values of the resistance of the lower wire B represented by the portion of the curve W, Y, Z. A consideration of these portions of the resistance curves in Fig. 5 leads to the conclusion that the free convection current from the wire B is of



approximately uniform temperature over a width $d \sin 10°$ on either side of the wire, where d is the distance apart of the two wires. In the present case, $d=0{\cdot}10$ cm. $2d \sin 10°$ may be termed the effective width of the free convection current, and is equal to 0·034 cm., which is of the same order of magnitude as the diameter of the wire employed. Heat is therefore directly conveyed to the ascending free convection current over a width corresponding to about three times the diameter of the wire. The width is doubtless greater than the diameter of the wire owing to the fact that the

wire is circular and not "stream line" in section. The various other points connected with the deflection inclination curves in Fig. 4 can be readily interpreted by similar reference to the resistance-inclination curves contained in Fig. 5. Reference may be made to the fact that there is close symmetry between the positive and negative portions of the deflection-inclination curves in Fig. 4, about the 90° and 270° positions respectively. There is not quite such symmetry between the 0° to 180° portion and the 180° to 360° portion of the same curves. This is doubtless due to slight want of symmetry in the disposition of the wires to one another, and to the walls of the chamber containing them. In Fig. 5 there is complete symmetry between the portions of the respective portions of the resistance-inclination curves included between the values 0° to 90° and 90° to 180°. Similar symmetry is seen between the 180° to 270° and the 270° to 360° portions of the several curves. The portions of the various resistance-inclination curves between the values 0° to 180° are quite unsymmetrical with regard to the 180° to 360° portions.

Referring now to the case in which the axis of rotation is at right angles to the plane of the wires, it is seen from Fig. 4 that the device possesses its maximum sensitivity in the neighbourhood of the origin. This is in accord with the approximate theory developed above, wherein it is shown that the increased heating effect to which the upper wire is subjected on rotation through an angle θ from its initial horizontal position is proportional to $(l-d\cot\theta)$. The cooling effect experienced by the lower wire, and hence the differential heating or cooling effect experienced by the two wires, is likewise proportional to the same expression. We may therefore write H, the differential cooling or heating effect in the form $H=\mu(l-d\cot\theta)$ and

$$\frac{dH}{d\theta}=\mu d\operatorname{cosec}^2\theta,$$

and this has its maximum values when $\theta=0°$, 180°, 360°; and its minimum values when $\theta=90°$ or 270°. The form of the curve in Fig. 5 is in approximate agreement with this theory, the curve being steepest for the values 0°, 180° and 360° of the inclination, and to a first approximation, flat in the region of the inclinations 90° and 270°. One outstanding feature of the resistance curves shown in Fig. 5 deserves mention. Whereas the sensitiveness of the arrangement wherein the axis of rota-

tion is parallel to and in the plane of the wires is due principally to the increased heating effect experienced by the upper wire of the pair, the sensitiveness of the other arrangement, in which the axis of rotation is at right angles to the plane of the wires, is due principally to the increased cooling effect experienced by the lower wire. This is made clear from a consideration of the portions of the various curves included within the region of inclinations 0° to 180° and 180° to 360°. The initial balance in the original horizontal position is effected under different conditions in the two cases. In the first case the horizontal axis of rotation, being parallel to the wires, each wire is cooled by the free convection current arising from it. In the other case, the wires being initially vertical, each is laved by its own free convection current, the resistances being therefore greater than when measured with both wires horizontal. With the wires initially vertical, it is seen from Fig. 5 that the maximum heating of the upper wire A occurs on rotation of the arm through 30° or 150° from the initial horizontal position. This arises from the fact that on initial rotation from the original horizontal position the heating of the wire A due to its own free convection current diminishes while heating due to incidence of the free convection current arising from B increases. Initially the temperature of A rises, but ultimately the cooling of A due to its own free convection current increases to such an extent as to equal exactly the heating effect due to incidence of the hot free convection current arising from B. This condition of affairs is represented by the point N in Fig. 5, which is such that the resistance of the wire A is equal to its initial resistance with the arm horizontally disposed. The resistance of the lower wire B is seen to decrease continuously during rotation of the arm through the first quadrant, and the approximate theory developed above is able to explain at least some of the facts, largely on account of the large magnitude of the cooling effect on the lower wire compared with the smaller thermal effects experienced by the upper wire. It is clear that if A and B were removed to a great distance from one another, the bridge initially balanced with A and B vertically disposed would remain balanced, all other things remaining the same, whatever the inclinations of A and B were, so long as they were the same. In the consideration of the deflection curves, therefore, auto-cooling of the wires due to their own free convection currents may to a first approximation be neglected, the necessary correction being merely that due to the

difference in the velocity and temperature of the respective free convection currents owing to the difference in temperature of the respective wires. The variation with temperature of the velocity of the free convection current is considered in detail in the Paper by the author already referred to.

The depressions shown at the points T and T' of the curve for the case when the axis of rotation is at right angles to the wires (Fig. 4) deserve notice. It has been already remarked that the final state as regards temperature of the respective wires is determined by the production of equilibrium conditions governed by considerations of heating of the respective wires by the electric current, by radiation from each wire, and of cooling or heating by convection currents arising from each wire. In the present case, using a heating current of 1·000 ampère, the equilibrium conditions represented by T are such (as seen from T, Fig. 5) that the differential heating effect in the two wires attains a minimum value when the arm has been rotated through 90°. A reference to T, Fig. 5, shows that in the neighbourhood of 90° the resistance of the upper wire falls more rapidly than that of the lower wire. The effect can readily be interpreted in the light of the conditions of equilibrium already referred to.* It might appear at first sight that the depression represented by T originated in thermoelectric effects in the wires, the ends of which are under different thermal conditions in the case where the axis of rotation is at right angles to the wires, except when the wires are so disposed that the ends of one are vertically above those of the other. It is, however, readily seen from the symmetry of the curve in the region of T that such an explanation is entirely untenable. Apart from the elimination of thermoelectric effects from the deflection by reversal of the bridge current, it is clear that if at a small angle, θ, on one side of the 90° position, the effect of unbalanced resistances in the bridge, and any possible thermo-electric effect be represented by E and e respectively, the resultant E.M.F. in the bridge being $E-e$, at the same angle θ on the other side of the 90° position, the E.M.F. in the bridge would be represented by $E+e$ and in the 90° position by E. On rotation, therefore, from the one position to the other, the resultant E.M.F. producing deflection progressively passes through the increasing series of values—$E-e$, E, $E+e$—with consequent lack of symmetry of the calibration curve about the 90° position. The depression at

* "Phil. Mag.," 1920, *loc. cit.*

T (Fig. 4), therefore, does not originate in thermo-electric effects. The depression is, apparently, not present in the curve Fig. 4, for which the axis of rotation is parallel to the wires. As its apparent absence from the curve might originate in its smaller magnitude in this case, and as, moreover, in this case the possibility of a thermo-electric origin is definitely ruled out, owing to the symmetrical disposition of the ends of each wire with regard to those of the other, it seemed desirable to investigate the possible existence of the effect when the wires were parallel to the axis of rotation. For this

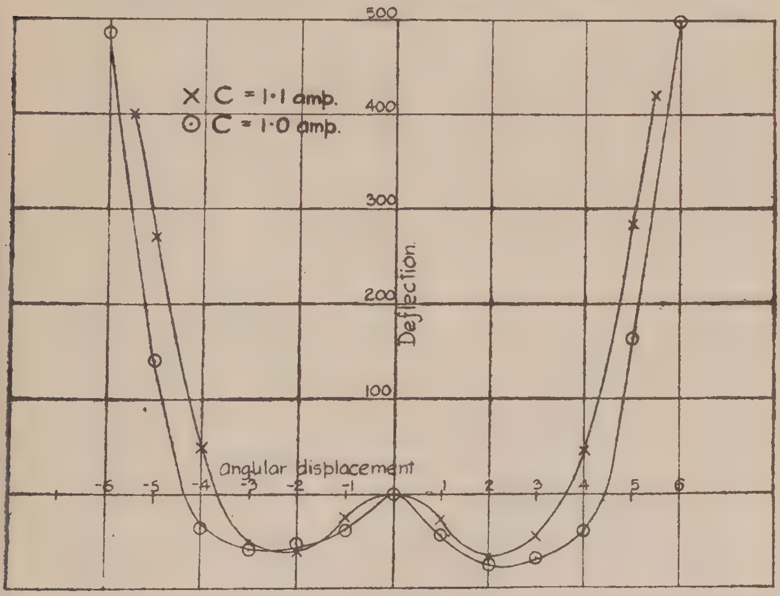
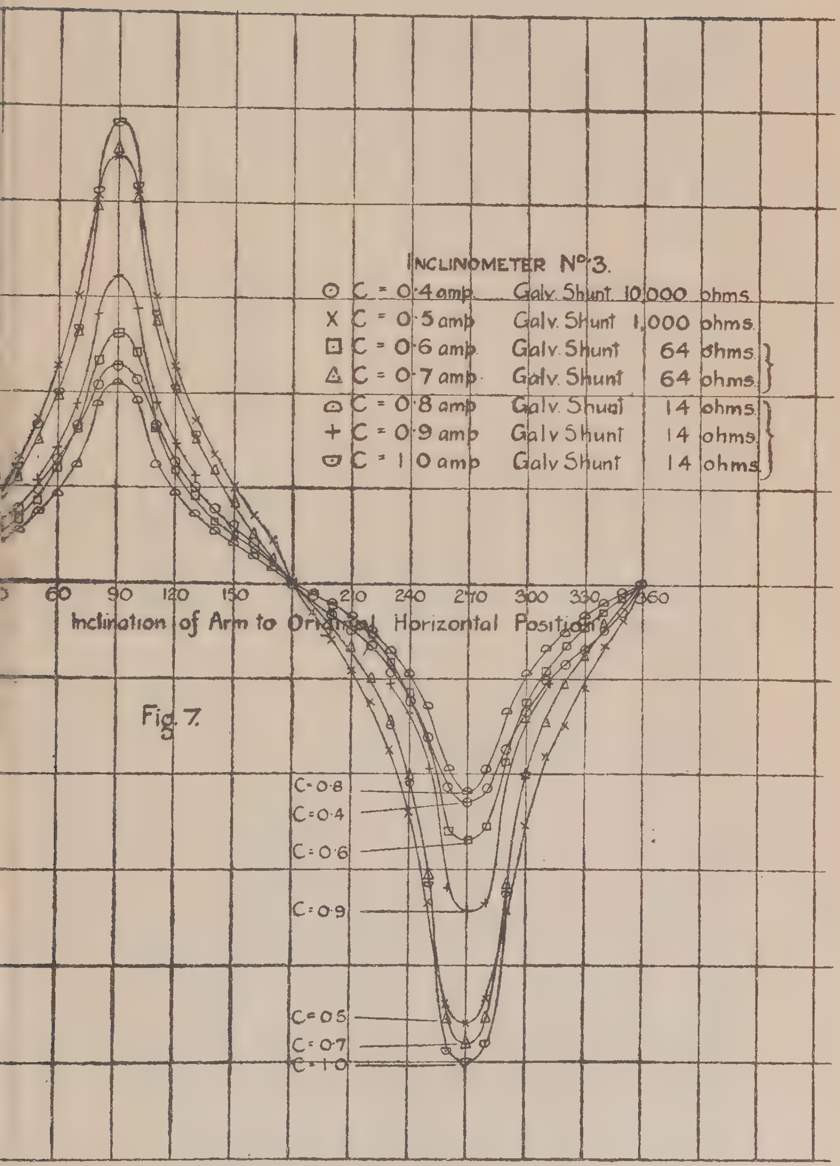


Fig. 6.

purpose the bridge was balanced with the arm rotated through 90° from the horizontal position—*i.e.*, with the one wire *A* vertically above the other *B*. The deflections when the arm was rotated through successive angles of 1° on either side of this zero position were then read, the current in the bridge being maintained constant at 1·000 ampère. The galvanometer was employed at its maximum sensitivity. The fixed arm of the bridge was adjusted to 1,000 ohms, and exact balance of the bridge with the arm in the vertical position was obtained by unplugging resistances from a box inserted in parallel with

that portion of the adjustable arm of the bridge containing coils of resistance 1, 2, 3 and 4 ohms. The drop of potential across one of the wires was measured, and the bridge was balanced with the arm in each of the various positions, thus enabling the values of the resistances of the two wires to be determined. The variations of the respective resistances with rotation of the arm through a maximum of 6° were small, at the most of the order of 0·0008 ohm. The values of the deflections are plotted in Fig. 6 as ordinates against the angular displacements of the arm from the vertical position as abscissæ. Results obtained employing a bridge current equal to 1·1 ampère, the galvanometer being shunted with 1,000 ohms, are similarly given in Fig. 6. The results show that the depression, although confined to narrower limits than in the case already referred to, is present in the calibration curve for the case when the axis of rotation is parallel and in the plane of the wires. The deflection, initially negative, is seen to be reversed when an angular displacement of 4° in either direction from the vertical position is reached. Thereafter the deflection becomes increasingly positive. The existence of this depression in the calibration curve somewhat restricts the use of the inclinometer No. 2 in the region of about 5° on either side of the vertical position. By suitable choice of current, and distance between the wires, the depression in the calibration curve can be entirely eliminated. Thus, with inclinometer No. 3, employing a bridge current of 0·7 ampere, the deflection *continuously* increased as the arm was rotated from an inclination of 80° with the horizontal to the vertical position, thereafter decreasing *continuously* as the arm was rotated through an angle of 10° on the other side of the vertical, the deflections being measured at successive intervals of 1°. Fig. 7 gives the forms of the calibration curves of inclinometer No. 3, employing various bridge currents between 0·4 ampere and 1·0 ampere. The temperature to which the wire further from the axis of rotation was raised in the zero position by the respective heating currents employed is given in Table III. being deduced from the resistance and temperature coefficient of the wire in the usual manner. The temperature of the wire nearer the axis of rotation was initially very approximately the same. The axis of rotation was in every case parallel to and in the plane of the wires. The galvanometer was suitably shunted in each case, and the values of the respective shunts are indicated in Fig. 7.



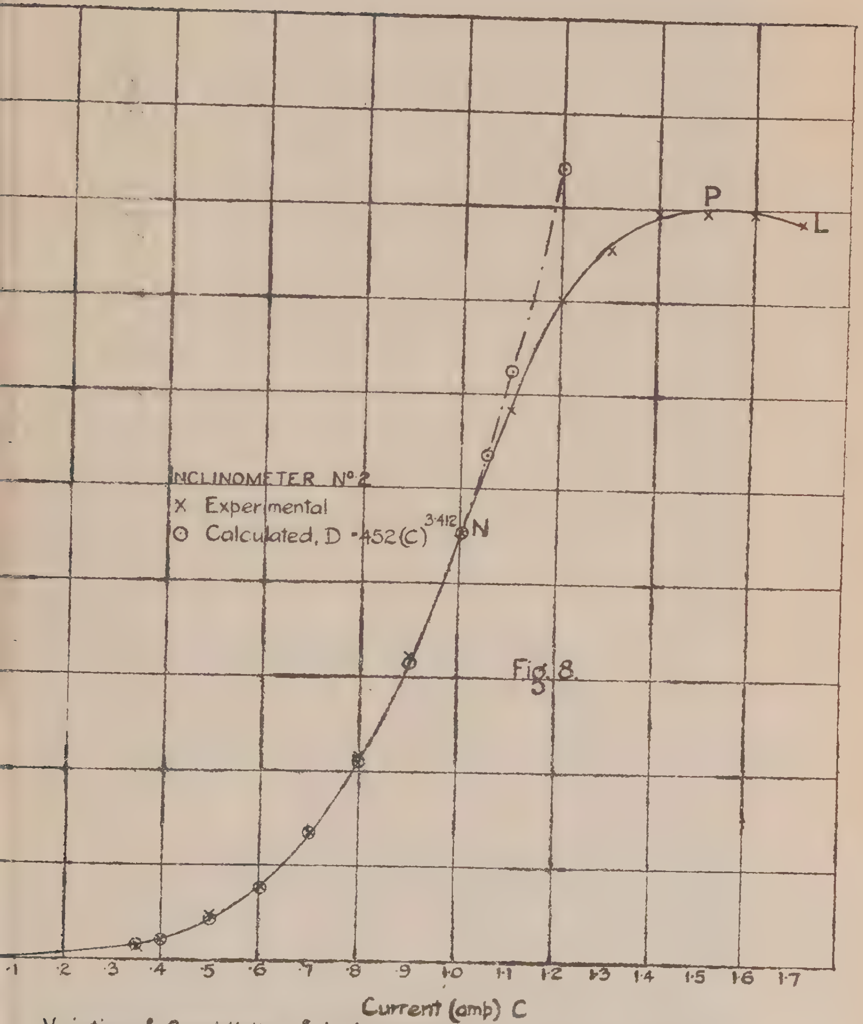
Axis in plane of and parallel to wires.

The use of small heating currents reduces the expansion of the wires, and on this account is to be preferred, provided sufficient sensitiveness of the galvanometer can be secured, a matter generally of very little difficulty. In Fig. 7 the curves are not all strictly comparable with one another, as the galvanometer shunt employed was not the same in all cases. The curves for values of the current equal to 0·6 and 0·7 ampère are strictly comparable one with another, as are likewise the curves for values of the current equal to 0·8, 0·9 and 1·0 ampère.

TABLE III.

Heating current.		Temperature of wire.
0·4	....	44°C.
0·5	....	67°.
0·6	....	92°.
0·7	....	135°.
0·8	....	186°.
0·9	....	245°
1·0	....	330°

Fig. 8 shows how the deflection produced on rotation of the inclinometer through 90° from the horizontal to a vertical position depends upon the magnitude of the bridge current employed, the axis of rotation being parallel to and in the plane of the wires. The deflection is seen to attain its maximum value for a value of the bridge current equal to about 1·5 ampere, thereafter decreasing slightly as the current is increased. Fusion of the upper wire occurred when a bridge current equal to 1·8 ampere was used. The gradient of what may be termed the current-sensitivity curve is seen to diminish for values of the bridge current corresponding to about 1·0 ampere. The author has previously shown that, corresponding to a temperature of about 300°C.—which is approximately the temperature to which the platinum wire is heated by a current of 1 ampere (*see* Table III.)—the velocity of the free convection current from the wire is about 7 cm. per second, assuming the temperature of the free convection current to be that of the wire. In the Paper already referred to, dealing with a directional type of hot-wire anemometer, it was shown that the deflection diminished when the impressed velocity of the air current attained a value of about 4cm. per second. The wires in the directional anemometer being disposed horizontally, the effective cooling velocity corresponding thereto is about 8 cm. per second. While the effective cooling current of air is, in general, differently disposed with regard



Variation of Sensitivity of Inclinometer with Current in Bridge

to the plane of the wires in the case of the directional anemometer and the inclinometer, there can be no doubt that the falling off in the sensitivity of the latter for the higher values of the heating current is to be attributed to the same cause as that operative in the case of the former. The increasing velocity of the free convection current from the lower wire is accompanied by a diminished temperature to which the current of air is raised. The subsequent fall of temperature occurring in transit of the air from the lower to the upper wire is such that the differential thermal effect on the upper and lower wires respectively decreases as the heating current is increased beyond the value corresponding to the point P. The diminished gradient of the sensitivity curves for values of the bridge current greater than 1·0 ampère can be similarly explained. Its form is conditioned by two opposing tendencies—viz., the rise of temperature to which the free convection current of air is heated by passage over the lower wire and the subsequent fall of temperature in transit from one wire to the other. A useful analogy is afforded by comparison of the portion NPL of the curve with the familiar current-voltage curve in the case of the conduction of electricity through an ionised gas, where similarly two opposing tendencies are operative—viz., the increased velocity of the ions under increased potential gradient and the increased re-combination of the ions accompanying the same. For values of the bridge current included between 0·35 ampère and 1·05 ampère, the values of the deflection D on rotation through 90° from horizontal position are seen to be very accurately represented by the relation $D=452\ (C)^{3.412}$, where C is the bridge current employed, measured in amperes (*see* Fig. 8). The existence of a point of inflexion in the curve in the neighbourhood of N, indicates the possibility of eliminating the depression in the calibration curve in the neighbourhood of 90° and 270° (*see* Fig. 4) by the use of a smaller current in the bridge. The curve in Fig. 8 may be usefully compared with that for the directional type of anemometer given in the Paper already referred to.*

The double-wire type of inclinometer described may be employed to afford optical evidence of the rotation of a body about a horizontal axis. For this purpose the Wheatstone bridge can be dispensed with, and the two wires merely joined

* *See* Proc. Phys. Soc., XXXII., p. 200, 1920.

in series with a battery and rheostat for adjustment of the current in the wires to a predetermined value, as indicated by an ammeter inserted in the circuit. The wires are viewed against a dark background. Thus, in the case of inclinometer No. 3 described above, using a current equal to 1·175 ampère, the wires being horizontally disposed one above the other, the upper wire was a very bright red, while the lower was just discernible. With rotation of the plane of the wires about a horizontal axis perpendicular to the plane of the wires, the relative brightnesses of the wires became more nearly equal, until apparent equality was reached when the wires were vertical. Thereafter, on further rotation in the same direction, the wire which was initially the lower of the pair became the higher, and consequently glowed more brightly than the other. Such a pair of wires, therefore, employed with the appropriate heating current would enable the upright or inverted disposition of an aeroplane to be readily ascertained when it is surrounded by clouds precluding the possibility of viewing the earth's surface. The device should find numerous applications in other directions, as it embodies an apparatus for measuring angles whose sensitiveness is readily adjusted within the extremely wide limits of sensitivity of the Wheatstone bridge and galvanometer employed. Thus, in Fig. 6, rotation through 1° corresponds to a deflection of about 350 mm.

The foregoing research arose out of an extensive research into the possibilities of hot-wire anemometry carried out at the physical laboratory of the South Metropolitan Gas Company, and the author desires to express his deep gratitude to Dr. Charles Carpenter for his unfailing readiness to afford all facilities for carrying out the work.

ABSTRACT.

The hot-wire inclinometer consists essentially of two fine platinum wires (diameter about 0·1 mm.) mounted parallel to one another in a closed chamber at a distance apart equal to about 1 mm. A constant current of from 0·4 to 1·5 ampere (according to the sensitivity desired) is maintained in the wires, which constitute two of the arms of a Wheatstone bridge arranged so that the bridge is balanced in the zero position of the inclinometer. If desired, the bridge may be constituted of two pairs of heated wires as above. The inclinometer wires, in the zero position, may be either vertical or horizontal or inclined at any angle to the horizontal. The indications of the instrument are dependent upon the relative heating or cooling effects experienced by the respective wires when the orientation of the wires

with regard to a horizontal plane is altered. The galvanometer deflection occurring with alteration in such orientation serves to indicate the inclination of a definitive radius vector to the horizontal, and calibration curves are given for the cases where the axis of rotation is horizontal, and (*a*) in the plane of the wires and parallel to the wires, and (*b*) at right angles to the plane of the wires. The characteristics of the calibration curves are discussed and curves are given showing how the resistances of the respective wires depend upon the orientation of the wires with regard to the horizontal plane referred to. Attention is directed to the finite width of the column of heated gas ascending from the wires, and an empirical formula is deduced expressing the maximum deflection as a function of the bridge current employed.

DISCUSSION.

Dr. Barratt mentioned that in a previous Paper Mr. Thomas had attributed certain effects to radiation between wires separated by about 100 times their diameter. He (the speaker) had pointed out that such an effect would be entirely negligible; and he was glad to see that the author had discarded radiation in the present Paper.

Mr. Gosling said that Mr. Thomas's apparatus seemed excellent for exploring the temperature in the convection stream both above and below a hot wire. He was not clear whether Mr. Thomas assumed that the narrow column of rapidly moving air extended to an equal extent below the hot wire. It would be interesting to establish if this were the case. Another interesting point was the connection, if any, between the central core of the hot column and the total width. When a hot body is standing in air there is a very definite zone of hot gas surrounding it. He had observed this recently by a modified Töpler "schlieren" method (which he described), but it was not possible to say from the appearance of the currents above a candle-flame, for instance, whether an actual discontinuity existed at the edge of the curent or not.

Mr. J. Guild pointed out that these and other points could readily be investigated quantitatively by placing the apparatus in one beam of a Michelson interferometer arranged to give "Contour" fringes.

Mr. F. J. W. Whipple referred to the practical applications such as the use of the instrument in an aeroplane. How would the results be effected by angular acceleration of the system?

Mr. F. E. Smith emphasised this point. It appeared that this inclinometer would give precisely the same indication as a mechanical inclinometer, such, for instance, as a suspended bob or a marble in a glass cylinder. In the two possible cases of unaccelerated flight, viz., right way up and upside down, there was never any doubt.

Dr. Barlow asked if the pressure of the gas affected the results given by the instrument?

Dr. Hopwood asked how the behaviour varied with the diameter of wire used? Some time ago he had heated two loops of fine wire in series in coal gas. At atmospheric pressure the finer of the two glowed perceptibly and the other did not. On reducing the pressure it was possible to get the thicker wire glowing more brightly than the other. Could any anomalous effects arise from such causes as this?

Mr. Thomas, in reply to Dr. Barratt, remarked that in the previous Paper referred to, the observed effects were not attributed wholly to the effect of radiation. Radiation was mentioned as playing some part, possibly a very minor part, in the interpretation of the phenomena referred to therein, and, contrary to the view expressed by Dr. Barratt as regards omission of reference to the effect of radiation in the present Paper, the Author felt that

the radiation loss should be at least mentioned in the present connection. Conditions could readily be imagined under which radiation losses, small though they admittedly are compared with convection losses, might exercise an important rôle in the attainment of equilibrium conditions by the wires. The radiation losses from the respective wires are readily calculable from the results of Lummer and Kurlbaum's observations (Ver. Phys. Ges., Berlin, 1898, 17, 106). For polished platinum, the radiation in watts per square centimetre is given by $e=0{\cdot}514\ (\theta/1{,}000)^{5 \cdot 2}$. King (" Phil. Trans.," 1914, A 520, 422) gives a useful table of the respective radiation and convection losses from fine platinum wires of diameters between 0·0028 cm. and 0·0153 cm. for various temperatures ranging from 145°C. to 1,179°C. It is interesting to note that King, in another Paper ("Phil. Mag.," 1915, 29, 576), refers to what he terms " kinetic heating "—the heat transfer to a fine platinum wire by impact of the gas stream. This effect is normally extremely small. In reply to Mr. Gosling, the Author stated that the results obtained, more particularly those indicated in Figs. 4 and 5, pointed to the existence of what amounted to a sharp discontinuity in the ascending column of heated gas from the wire. The distribution of temperature and velocity in the convection current surrounding a heated fine platinum wire was at present being investigated. The probability is that a sharp discontinuity does not exist in the ascending column of gas below the heated wire. In a recent Paper (" Phil. Mag.," 1920, 39, 533) the Author had detailed measurements of the ratios of the thermal conductivities of various gases to that of air by comparison of the forced convection losses of heat from a heated wire in the respective gases. The wire was at a temperature of about 350°C., but the ratio of thermal conductivities found was that at about atmospheric temperature. This was interpreted as affording evidence of the existence of a skin or film of gas round the wire, which served as a medium for conducting heat from the wire to the stream of gas. Such a film possibly plays no inconsiderable part in determining the width of the hot column of gas ascending from the wire already referred to. It also affords a physical interpretation of the " effective " radius of the wire occurring in certain formulæ for the convection heat loss. The existence of such a film is supported by the results obtained by Langmuir (" Proc." Amer. Inst. Elec. Eng., 31, 1011-1022 ; " Phys. Rev.," May, 1912, 34 ; " Trans." Amer. Electrochem. Soc., 1913, 23, 293). The interferometer method suggested by Mr. Guild appeared to be admirably suited to the investigation of the temperature distribution in the ascending column of gas. The method would not, apparently, lend itself to the *direct* determination of the velocity distribution, though useful information under this head would doubtless be afforded by a knowledge of the temperature distribution. With regard to the remarks of Mr. Whipple and Mr. Smith, the Author agreed that the instrument indicated only the *apparent* direction of gravity as affected by centrifugal and other acceleration forces. He was, however, under the impression, which seemed to be rather a common one, that no means existed for discriminating between " right way up " and " wrong way up " in certain cases, and that no satisfactory indicator for the purpose existed. The device could be readily modified so as to indicate rotation about a vertical axis, and in this form might conceivably be usefully employed in experiments in wind channels for setting models accurately at any desired inclination to the wind current. The device possessed the advantage that it enabled inclinations to be indicated or recorded at any station however distant from the experimental station. The device described in the Paper enabled one to discriminate angles situated in the first and second quadrants from those in the third and fourth. A modification in which the wires were in the same horizontal plane when the radius vector was vertical, used in conjunction with the device described, would enable the actual quadrant to be uniquely determined. In reply to Dr. Hopwood, the

indications afforded by inclinometers constructed of wires up to 0·2 mm. were very similar, although their sensitivities were somewhat different. Experiments in this direction are being extended at present. No anomalous effects of the nature referred to by Dr. Hopwood had been noticed in the course of the present experiments, in which the wires were throughout surrounded by air. The use of the wires in coal-gas, particularly if they are employed at even a moderately elevated temperature, is accompanied by the introduction of difficulties due to occlusion of coal-gas by the wire, and possibly by decomposition of certain hydro-carbon constituents of the gas. Such possibilities may have some bearing on the interpretation of the phenomenon observed by Dr. Hopwood. In reply to Dr. Barlow, the Author stated that the experiments had hitherto been confined to air at atmospheric pressure.

XXVIII. *The Radiation from a Perfectly Diffusing Circular Disc. (Part II.)* By John W. T. Walsh, *M.A.*, *M.Sc.* (*From the National Physical Laboratory.*)

In a previous Paper* an expression was obtained for the amount of flux received from a perfectly diffusing circular disc (*i.e.*, one radiating according to the cosine law) by another parallel and coaxial disc of given diameter, placed at a given distance from it. The method was also extended to determine the amount of flux received by the sides of an infinitely long coaxial circular cylinder bounded by a disc parallel to the radiating disc.

In the present Paper the results already obtained will be applied to give (i.) the amount of flux reflected to any point of the radiating disc if the non-radiating disc reflect the flux incident upon it according to the cosine law, and (ii.) the flux received at any point in the penumbra of the shadow formed when a radiating disc is partially eclipsed by a second coaxial opaque disc.

A treatment will also be given of the problem of finding the flux incident upon a disc inclined at an angle with the radiating disc. So far, the accurate solution of this problem has only been obtained when the edges of both discs form small circles of the same sphere (Part I., p. 65). A solution in the form of an infinite series will be obtained for the less specialised case when the axis of the receiving disc passes through the centre of the radiating disc, the two discs being unrestricted as to relative size.

(VII.) *The amount of flux reflected back to a radiating circular disc by a coaxial diffusely reflecting circular disc of given coefficient of reflection.* Let EF (Fig. 1) be a radiating circular disc of radius R, and let the flux emitted in the normal direction per unit area be Φ. Then if B be an element of a coaxial disc CD of radius r, the flux falling on B is

$$\frac{B}{2\pi r}\cdot\frac{d}{dr}\left[\frac{\pi^2\Phi}{2}(a^2+r^2+R^2-Q_r)\right]$$
$$=\tfrac{1}{2}\Phi B\pi[1-(a^2+r^2R^2)/Q\ ],$$

where $$Q_r=\sqrt{(a^2+r^2+R^2)^2-4r^2R^2},$$

* "The Radiation from a Perfectly Diffusing Circular Disc (Part I.)," Proc. Phys. Soc., XXXII., 1920, p. 59.

and hence the flux received per unit area by a ring, of depth dr and radius x, is

$$\tfrac{1}{2}\Phi\pi[1-(a^2+x^2-R^2)/Q_x].$$

Therefore the flux radiated by this ring in the normal direction is, if k be the coefficient of reflection of the disc CD,

$$\tfrac{1}{2}k\Phi[1-(a^2+x^2-R^2)/Q_x].$$

Hence the flux received back from this ring by the disc EF is (by Section II. Note),

$$k\Phi\pi x dx[1-(a^2+x^2-R^2)/Q_x]\times\tfrac{1}{2}\pi[1-(a^2+x^2-R^2)/Q_x].$$

Hence the total flux received back is

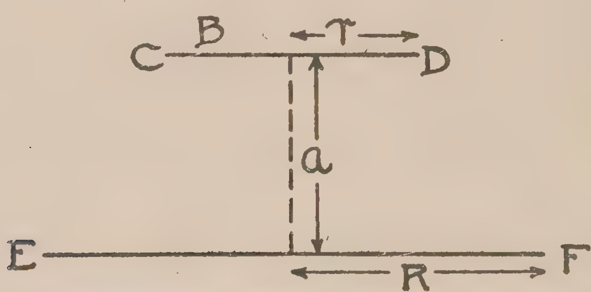


Fig. 1.—Flux Reflected Back to Radiating Disc by Coaxial Reflecting Disc.

$$\tfrac{1}{2}k\Phi\pi^2\int_0^r[1-(a^2+x^2-R^2)/Q_x]^2dx.$$

Let $a^2-R^2+x^2=2aR\tan\alpha$, then the above expression reduces to $\qquad \tfrac{1}{2}k\Phi\pi^2 aR\int(\sec\alpha-\tan\alpha)^2d\alpha,$

the limits of integration being

$$\tan^{-1}\frac{a^2-R^2}{2aR}$$

and $\qquad \tan^{-1}\dfrac{a^2+r^2-R^2}{2aR}$

$$=\tfrac{1}{2}k\Phi\pi^2\left[(a^2+r^2+R^2)-\sqrt{(a^2+r^2+R^2)^2-4r^2R^2}\right.$$
$$\left.-aR\tan^{-1}\frac{2ar^2R}{(a^2-R^2)r^2+(a^2+R^2)^2}\right]$$

As a numerical example, suppose $a=R=2r=1$. The above expression reduces to $\tfrac{1}{2}k\Phi\pi^2(0\cdot1103)$. The total amount of

light radiated from the disc is $\pi^2 \Phi R^2$, so that the percentage reflected back is $k(5{\cdot}51)$ per cent.

The percentage of the radiated flux which is received by the reflecting disc is, in this case, 11·72 per cent., so that 47·01 per cent. of the flux received by the smaller disc is returned by it to the larger, if the value of k is unity.

If the discs are of equal size ($a=R=r=1$) the flux received on the reflecting disc is 38·2 per cent. of the total radiated, and the flux received back by reflection (supposing $k=1$) is 15·0 per cent. of that radiated. It is clear that this reflected flux will rapidly diminish as a increases and Fig. 2 shows graphi-

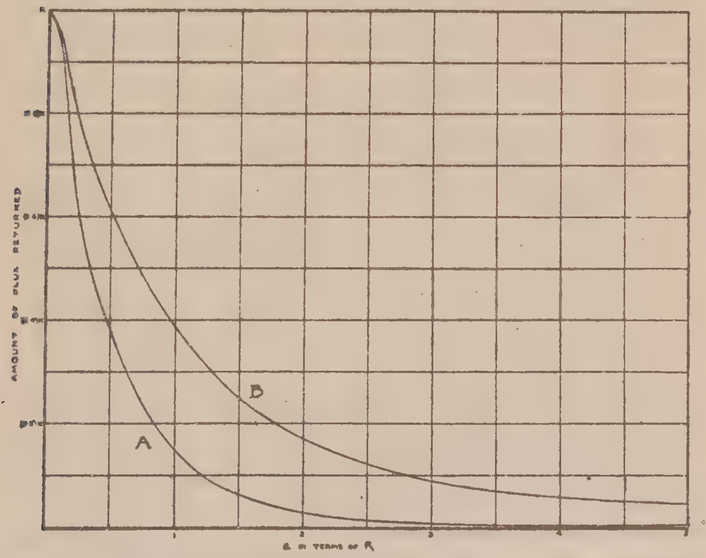


Fig. 2.—Amount of Flux Returned to Equal Coaxial Reflecting Disc at Various Separations, a.

Curve A.—Flux received back by radiating disc.
Curve B.—Flux received by reflecting disc.

cally the value of this reflected flux for different values of a supposing $r=R$.

(VIII.) *The flux received by a point in the penumbra of the shadow of an opaque disc cast by a coaxial radiating disc.* In Fig. 3, let AB be the radiating disc, of radius E, and CD a coaxial opaque disc of radius r. Let O be the point at which it is required to find the flux incident from AB.

Let the circles whose centres are L and M represent the disc AB and the projection from O of the disc CD on the plane of AB. Also let O' represent the orthogonal projection of O on this plane. Then if a and b are the dis-

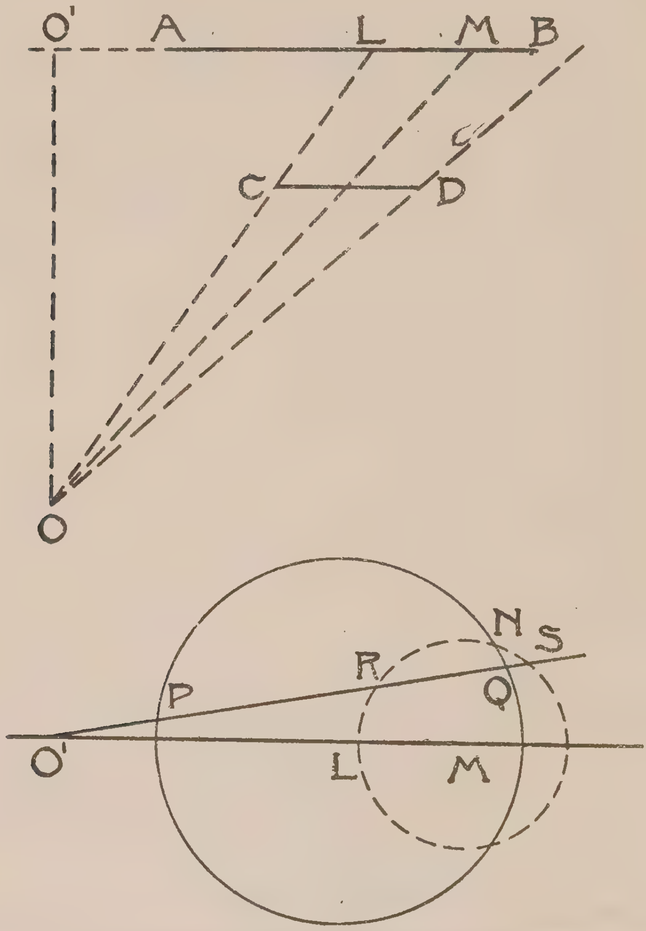


FIG. 3.—FLUX RECEIVED IN PENUMBRA OF SHADOW FORMED BY TWO COAXIAL DISCS, ONE RADIATING AND ONE OPAQUE.

tances of O from the planes of AB and CD, the effective radius of the eclipsing disc will be ar/b. Let a line be drawn through O' cutting the circles in P, Q, R and S, as shown in the figure. Then, if B be the area of the element of surface at O, and Φ the

normal radiation from the disc, the flux eclipsed is (*see* Part I., section I.)

$$B\Phi\int_0^\theta\left[\frac{OQ^2}{OQ^2+a^2}-\frac{OR^2}{OR^2+a^2}\right]d\alpha,$$

where $$\theta=\angle LON.$$

Now

$$\int_0^\theta\frac{OQ^2}{OQ^2+a^2}d\alpha=\tfrac{1}{2}\int_0^\theta\left(\frac{OQ^2}{OQ^2+a^2}+\frac{OP^2}{OP^2+a^2}\right)d\alpha+$$
$$\tfrac{1}{2}\int_0^\theta\left(\frac{OQ^2}{OQ^2+a^2}-\frac{OP^2}{OP^2+a^2}\right)d\alpha,$$

and, by Part I., section VI.,

$$\int_0^\theta\left(\frac{OQ^2}{OQ^2+a^2}+\frac{OP^2}{OP^2+a^2}\right)d\alpha=\left[\alpha+\frac{t^2+a^2}{F}\tan^{-1}\frac{(t^2-a^2)\tan\alpha}{F}\right]_0^\theta,$$

where

$$F^2=4a^2\rho_1^{\,2}+(t^2-a^2)^2$$
$$\rho_1=O'L \text{ and } t^2=\rho_1^{\,2}-R^2.$$

Also, by Part I., section II.,

$$\int_0^\theta\left(\frac{OQ^2}{OQ^2+a^2}-\frac{OP^2}{OP^2+a^2}\right)d\alpha$$
$$=\left[\sin^{-1}\frac{x}{R}-\frac{c}{\sqrt{c^2+R^2}}\cos^{-1}\frac{c}{R}\sqrt{\frac{R^2-x^2}{c^2+x^2}}\right]_{\sqrt{R^2-\rho_1^{\,2}\sin^2\theta}}^{R}$$
$$=\frac{\pi}{2}\left(1-\frac{c}{\sqrt{c^2+R^2}}\right)-\varphi+\frac{c}{\sqrt{c^2+R^2}}\cos^{-1}\sqrt{\frac{c^2\cos^2\varphi}{c^2+R^2\sin^2\varphi}},$$

where $$R^2\sin^2\varphi=R^2-\rho_1^{\,2}\sin^2\theta.$$

Similarly $\int_0^\theta\frac{OR^2}{OR^2+a^2}d\alpha$ is obtained, and hence the flux eclipsed is

$$\tfrac{1}{2}B\Phi\left[\frac{t^2+a^2}{F}\tan^{-1}\frac{(t^2-a^2)\tan\theta}{F}+\frac{c}{\sqrt{c^2+R^2}}\cos^{-1}\sqrt{\frac{c^2\cos^2\varphi}{c^2+R^2\sin^2\varphi}}\right.$$
$$-\varphi-\frac{\pi}{2}\left(\frac{c}{\sqrt{c^2+R^2}}-\frac{c'}{\sqrt{c'^2+R^2}}\right)-\frac{t'^2+a^2}{G}\tan^{-1}\frac{(t'^2-a^2)\tan\theta}{G}$$
$$\left.+\frac{c'}{\sqrt{c'^2+R^2}}\cos^{-1}\sqrt{\frac{c'^2\cos^2\varphi'}{c'^2+R'^2\sin^2\varphi'}}-\varphi'\right],$$

where $$t'^2 = \frac{a^2(\rho_1{}^2 - r^2)}{b^2}$$

and $$G^2 = 4a^2\frac{a^2\rho_1{}^2}{b^2} + (t'^2 - a^2)^2.$$

Also $$c = (a^2 - R^2 + \rho_1{}^2)/2a$$

and $$c' = (a^2/b^2)(b^2 - r^2 + \rho_1{}^2)/2a$$

$$R^2\sin^2\varphi = R^2 - \rho_1{}^2\sin^2\theta$$

and $$(a^2r^2/b^2)\sin^2\varphi' = (a^2r^2/b^2) - (a^2\rho_1{}^2/b^2)\sin^2\theta,$$

so that $$r^2\sin^2\varphi' = r^2 - \rho_1{}^2\sin^2\theta.$$

As an example, let $R=1$, $a=2$, $b=1$, $\rho_1=1$, $r=0{\cdot}5$,

then $t=0$, $\theta=30°$, $F=4\sqrt{2}$, $t'=3$, $G=\sqrt{89}$, $C=1$, $C'=7/4$, $\varphi=60°$, and $\varphi'=O$.

$\therefore$ Flux eclipsed $=\frac{1}{2}B\Phi(0{\cdot}179)$.

The flux received, supposing the eclipsing disc absent, is (by Section II.)

$$\tfrac{1}{2}\pi B\Phi[1 - c/\sqrt{c^2+R^2}] = \tfrac{1}{2}B\Phi(0{\cdot}92).$$

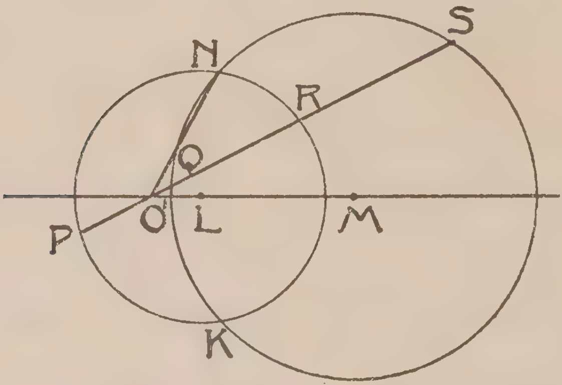


FIG. 4.—SECOND CASE OF PROBLEM SHOWN IN FIG. 3.

Thus, approximately 19·5 per cent. of the total flux is eclipsed in this case, and the illumination at the point considered is only 80·5 per cent. of its full value, supposing the eclipsing disc removed.

It will be noticed that in the above investigation it is assumed that the second point of intersection of the line *ON* (Fig. 1) with the circle whose centre is *M* lies outside the circle whose centre is *L*. If this be not the case, as in Fig. 4, the

problem may be attacked in an exactly analogous fashion, but substracting from the flux received from the uneclipsed disc PNR, the flux which would be received by a disc QNS of equal normal radiation per unit area. To the result must be added the flux received at O by the lenticular portion $NSKR$ of this latter disc. This portion is equal to

$$\int_0^\theta \left(\frac{OS^2}{OS^2+a^2} - \frac{OR^2}{OR^2+a^2} \right) da,$$

and

$$\int_0^\theta \frac{OS^2}{OS^2+a^2} da = \tfrac{1}{2}\int_0^\theta \left(\frac{OS^2}{OS^2+a^2} + \frac{OQ^2}{OQ^2+a^2} \right) da + \tfrac{1}{2}\int_0^\theta \left(\frac{OS^2}{OS^2+a^2} - \frac{OQ^2}{OQ^2+a^2} \right) da,$$

and $\int_0^\theta \frac{OR^2}{OR^2+a^2} da$ is obtained as before, so that the problem is solved in this case as well.

(IX.) *The flux received from a radiating disc by a second disc neither coaxial nor parallel with it, but such that its axis passes through the centre of the radiating disc.* Let Z be the zone of a

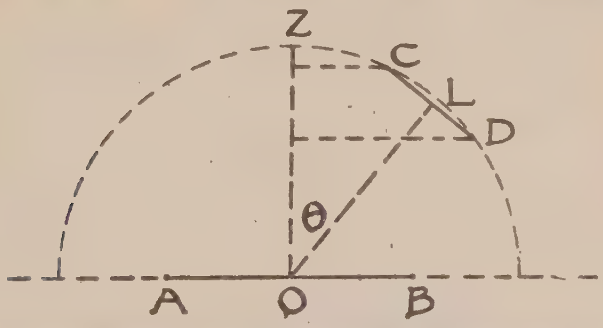


FIG. 5.—FLUX RECEIVED BY NON-COAXIAL DISC CD FROM RADIATING DISC AB.

hemisphere erected on the plane of the radiating disc AB, and passing through the edge of the receiving disc CD. Let OL be the axis of CD, passing through O, the centre of AB, and making an angle θ with OZ.

Now suppose CD divided into zones by great circles, such as ZPQ, passing through Z and cutting CD in P and Q, and let the angle LZP be a.

The $\widehat{ZP}$ and $\widehat{ZQ}$ are given by the quadratic equation

$$\cos \alpha = \frac{\cos r - \cos x \cos \widehat{ZL}}{\sin x \sin \widehat{ZL}}$$

where r is the angular radius of the disc CD (in terms of OZ).

Putting $\widehat{ZL} = \theta$ we have

$$\sin x = \{\cos r \sin \theta \cos \alpha \pm \cos \theta \sqrt{\sin^2 r - \sin^2\theta \sin^2\alpha} / (1 - \sin^2\theta \sin^2\alpha).$$

$$\therefore \quad \sin \widehat{ZP} + \sin \widehat{ZQ} = 2 \cos r \operatorname{cosec} \theta \cos \alpha / (\operatorname{cosec}^2\theta - \sin^2\alpha)$$

$$\sin \widehat{ZP} - \sin \widehat{ZQ} = 2 \cot \theta \sqrt{\operatorname{cosec}^2\theta \sin^2 r - \sin^2\alpha} / (\operatorname{cosec}^2\theta - \sin^2\alpha).$$

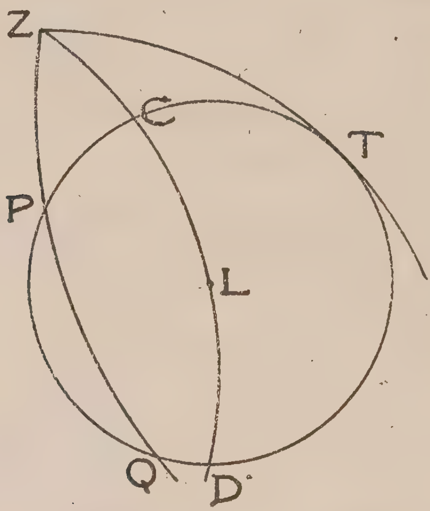


FIG. 6.—RECEIVING DISC CD CUT BY THE GREAT CIRCLE ZPQ.

Put $\sin r = l$, and $\sin \theta \sin \alpha = m$.

Then $\sin \widehat{ZP} + \sin \widehat{ZQ} = 2 \cos r \cos \alpha \sin \theta / (1 - m^2)$

$$\sin \widehat{ZP} - \sin \widehat{ZQ} = 2 \cos \theta (l^2 - m^2)^{\frac{1}{2}} (1 - m^2)^{-1}.$$

$$\therefore \quad \sin^2 \widehat{ZP} - \sin^2 \widehat{ZQ} = 4 \cos r \cos \theta \sin \theta \sin \alpha (l^2 - m^2)^{\frac{1}{2}}.$$

Now that portion of the total flux radiated from AB which is received by any zone of CD cut off by two such lines as ZPQ, the angle between the lines being $d\alpha$ is given by the expression $(1/2R^2)\{\sqrt{(R^2+1)^2 - 4R^2 \sin^2 a} - \sqrt{(R^2+1)^2 - 4R^2 \sin^2 b}\} (d\alpha/2\pi)$ where $a = \widehat{ZP}$ and $b = \widehat{ZQ}$, and $2R = AB/OZ$. Hence if τ be

the angle LZT, the flux received by the disc is

$$\{(R^2+1)/2\pi R^2\}\int_0^\tau [\{1-4R^2\sin^2 a/(R^2+1)^2\}^{\frac{1}{2}}-\{1-4R^2\sin^2 b/(R^2+1)^2\}^{\frac{1}{2}}]da$$

$$=(1/\pi\lambda R)\int_0^\tau [(1-\lambda^2\sin^2 a)^{\frac{1}{2}}-(1-\lambda^2\sin^2 b)^{\frac{1}{2}}]da$$

$$=(1/\pi\lambda R)\int_0^\tau \left[-\frac{\lambda^2}{2}(\sin^2 a-\sin^2 b)+\frac{\lambda^4}{8}(\sin^4 a-\sin^4 b)-\ .\ .\ .\right]da,$$

where $\lambda=2R/(R^2+1)$.

Now $(\sin^2 a-\sin^2 b)da=4\cos r\cos\theta(l^2-m^2)^{\frac{1}{2}}(1-m^2)^{-2}dm$

$$(\sin^4 a-\sin^4 b)/(\sin^2 a-\sin^2 b)=2(\cos^2\theta+\cos^2 r)(1-m^2)^{-1}-4\cos^2 r\cos^2\theta(1-m^2)^{-2}.$$

Also, the limits o to τ of a correspond with the limits o to l of m (for $\sin^2 r=\sin^2\theta\sin^2\tau$), so that $\int(\sin^{2n}a-\sin^{2n}b)da$ may always be expressed as $\int\Sigma f(r,\theta)(\sin^2 a-\sin^2 b)(1-m^2)^{-p}\,da$ where p is a positive integer and this expression

$$=\int_0^l \Sigma f(r,\theta)(l^2-m^2)^{\frac{1}{2}}(1-m^2)^{-p}\,dm,$$

which is always integrable for $A\int(1-m^2)^{-p}(l^2-m^2)^{\frac{1}{2}}dm$

$$=m(1-m^2)^{-p+1}(l^2-m^2)^{\frac{3}{2}}-B\int(1-m^2)^{-p+1}(l^2-m^2)^{\frac{1}{2}}dm-C\int(1-m^2)^{-p+2}(l^2-m^2)^{\frac{1}{2}}dm,$$

where $A=2(l^2-1)(p-1)$; $B=4(p-2)-l^2(2p-3)$; and $C=2(3-p)$. In this way it is found that

$$L_1=\int_0^l (l^2-m^2)^{\frac{1}{2}}(1-m^2)^{-2}=\frac{\pi l^2}{4}(1-l^2)^{-\frac{1}{2}}$$

$$L_2=\int_0^l (l^2-m^2)^{\frac{1}{2}}(1-m^2)^{-3}=\frac{\pi l^2}{16}(4-3l^2)(1-l^2)^{-\frac{3}{2}}$$

$$L_3=\int_0^l (l^2-m^2)^{\frac{1}{2}}(1-m^2)^{-4}=\frac{\pi l^2}{32}(8-12l^2+5l^4)(1-l^2)^{-\frac{5}{2}}$$

$$L_4=\int (l^2-m^2)^{\frac{1}{2}}(1-m^2)^{-5}=\frac{\pi l^2}{256}(64-144l^2+120l^4-35l^6)(1-l^2)^{-\frac{7}{2}},$$

$$L_5=\int_0^l (l^2-m^2)^{\frac{1}{2}}(1-m^2)^{-6}=\frac{\pi l^2}{512}(128-384l^2+480l^4-280l^6+63l^8)(1-l^2)^{-\frac{9}{2}}$$

and thus the values of $\int_0^\tau(\sin^2 a-\sin^2 b)da$, &c., can be calculated.

As a numerical example, take $l=\sin r=0{\cdot}4$ and $\sin\theta=0{\cdot}5$. Then it is found that

$$\int_0^{\tau}(\sin^2 a-\sin^2 b)da=(0{\cdot}1386)\pi$$

$$\int_0^{\tau}(\sin^4 a-\sin^4 b)da=(0{\cdot}07759)\pi$$

$$\int_0^{\tau}(\sin^6 a-\sin^6 b)da=(0{\cdot}0440)\pi.$$

Thus, if $R=1$, the amount of flux received by the small disc is

$$\frac{0{\cdot}1386}{2}+\frac{0{\cdot}07759}{8}+\frac{0{\cdot}0440}{16}+\ldots\ldots$$

$$=\frac{1}{100}\{6{\cdot}93+0{\cdot}97+0{\cdot}27_5+\ldots\ldots\}$$

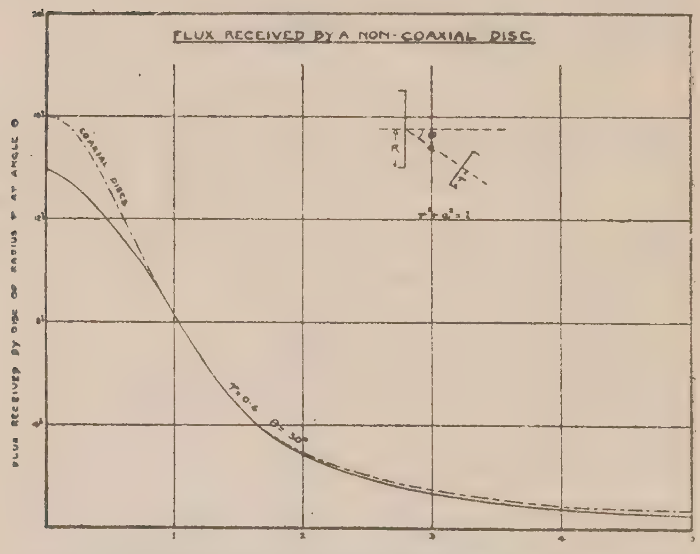


FIG. 7.—FLUX RECEIVED BY A NON-COAXIAL DISC FOR WHICH $r=0{\cdot}4$, $\theta=30°$, and $r^2+a^2=1$.

But, *in this particular case*, we know by the result obtained in Part I. of this Paper, that the total amount of flux received is $0{\cdot}083_5$, so that the sum of the residual terms in the above series is $0{\cdot}17_5$.

It is now easy to obtain the values of the above series for different values of R, for $\lambda=2R/(R^2+1)$ and the sum

$$=\frac{1}{100R}\{6{\cdot}93\lambda+0{\cdot}97\lambda^3+0{\cdot}27_5\lambda^5+\ldots\ldots\}$$

The values so obtained are shown graphically in Fig. 7, and superposed on this curve is the curve for the same values of r and R but with $\theta=0°$. It will be seen that the curve for coaxial discs lies above that for inclined discs everywhere, except, of course, at the value $R=1$, where they are coincident. The difference, however, is very slight except for small values of R, *i.e.*, when $R \lessapprox r$.

In the case of a searchlight arc, the values of R, r and θ, may be taken for the sake of example as 0·8, 0·4 and 30°, and for this case the obscuration is only 0·1 per cent. less than for axial carbons. Since the obscuration still takes place in the solid angle embraced by the mirror there would seem to be little advantage in this method of arranging the carbons unless either (i.) the negative carbon holder is very bulky, making R smaller than the effective value of r, or (ii.) is increased to bring the obscuring object outside the angle embraced by the searchlight mirror.

XXIX. *The Construction of Thermo-couples by Electro-deposition.* *By* WM. HAMILTON WILSON *and Miss* T. D. EPPS.

RECEIVED MAY 17, 1920.

ALTHOUGH thermo-electric appliances have been used in one form or another for nearly a century, the method of constructing these has remained practically unaltered up to the present time—namely, by joining together with solder two metals suitably related to one another in the thermo-electric series.

In practice this method has several serious drawbacks, which may be briefly stated as follows :—

(*a*) There is difficulty in forming reliable joints between the dissimilar metals, which will withstand working at high temperature without deterioration.

(*b*) Continuous use of such junctions is likely to lead to deterioration of the joints, resulting in a variable behaviour or a considerable increase in resistance, while if the joint has been formed by soldering the metals together the maximum working temperature is determined by the melting point of the solder.

(*c*) Owing to the extreme difficulty in making such joints between very small wires, it is impossible to reduce the mass of the metal at the points to the extent which is necessary for some special purposes.

(*d*) The labour and difficulty of constructing a large number of junctions to operate in series is excessive, and quite impracticable when the size of the wires becomes small.

In order to carry out a certain special research the need arose for some reliable and convenient means of constructing a line of a large number of junctions all in series and having small mass.

This was first attempted by joining together small wires of metals suitably placed in the thermo-electric series, and arranging them so that the alternate junctions to be heated should lie in a straight line, and that all the junctions should be connected in series.

The constructional difficulties when the number of junctions became large and of small mass, were found to be insuperable, although a number of methods was tried. Consequently, for a considerable period the idea of carrying out this research had to be abandoned, until the facility with which certain metals can be deposited electrolytically on others naturally suggested itself as a convenient means of uniting the metals selected.

In order to put such a method into practice, it is necessary first to prepare some form of conducting core or base upon which the metals to be used for the completed couples may be deposited electrolytically. This foundation should be capable of removal after it has served for depositing the metals of the couple, or it may be composed of some material of low conductivity relative to the conductivities of the deposited metals, so that its effect on their action is negligible even if not removed. Alternatively, the cross-sectional area of this base may be made so small relatively to those which the deposited metals are required to have that its effect is negligible.

For instance, if it is desired to form a thermo-couple of copper and iron, this may be done by taking a fine constantan wire which has a high specific resistance relatively to copper or iron, and coating one-half with copper, and the remaining half with iron to any desired thickness, so that the coatings of copper and iron come in contact at the middle point of the constantan core. Then, if the thicknesses of the copper and the iron coatings are sufficient, this will act for all practical purposes in a similar manner to a couple formed by joining together a copper and an iron wire in the ordinary way.

Obviously, this method is subject to a large number of variations, some of which would only be of use in special circumstances; for example, the core may be a film or fine filament of carbon, or it may be made of a platinum or silver film on glass, mica, &c.

Consideration of the effect of relative conductivities of the core and the metals employed for coating it, will show that with suitable precautions the core itself may be used to form one of the elements of the couple.*

In order to carry this into effect due regard must be paid to the cross-sectional area of the core where it is coated, relatively to that of the coating metal, taking into account their relative conductivities, in order that the presence of the core may not be prejudicial.

The first attempt to carry out this principle was made by depositing a sheathing of copper upon a constantan wire, half way along its length, as indicated in Fig. 1, with the intention that a thermo-E.M.F. should be set up at the point where the copper sheathing ceased, when this point was raised in temperature above that of the ends. It was expected that the

* Since writing the above a method giving similar results has come to our notice. See British Patent No. 19758 of 1904.

current would flow from the constantan wire to the copper sheath at the part heated, and that it would be practically uninfluenced by the presence of the constantan core. This was found to work successfully.

Then other experiments were made by using metals situated further apart in the thermo-electric series, such as an iron sheath on a constantan core, and also an antimony sheath on a constantan core. The results obtained with these metals were disappointing, owing to the smaller difference in the conductivity of the metals necessitating a much greater volume of metal in the sheath.

It is obvious that if the mass of the sheath has to be increased unduly in order to make the presence of the core negligible, the heat capacity of the couple is liable to become too great.

Fig. 1.-Plated Sheath.

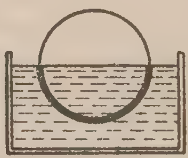
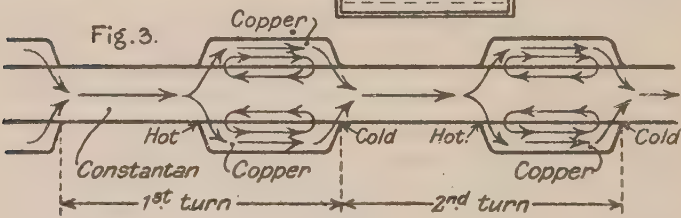


Fig. 2 Plating Bath.

Fig. 3.



After a good deal of experiment, constantan wires with either copper or silver sheaths were found to be suitable for most purposes.

Single junctions constructed by this method being found quite satisfactory, the next stage was to construct a pile consisting of lines of junctions in series. Two hundred spaced turns of 0·002 in. diameter constantan wire were wound upon a $\frac{1}{2}$ in. diameter ebonite rod. This was immersed in a copper plating bath, so that half the circumferential surface of the rod was above the level of the liquid. The wire was now plated, as in Fig. 2, thus forming a sheathing of copper on each turn for approximately half its length, the copper sheathings on all the turns ending more or less abruptly along lines diametrically opposite.

When the junctions thus formed between the constantan wire and the copper sheathings situated along one side of the cylinder were heated above the temperature of those on the opposite side, the contrivance behaved as a thermopile having a large number of junctions in series.

The path of the current in this composite conductor, at the positions corresponding to the hot and cold junctions, would be from the constantan wire to the copper sheath at the heated junction, and from the copper sheath to the constantan wire forming the next turn, at the subsequent cold junction, as shown in Fig. 3, in which the wire is represented as straightened out for clearness.

In addition to this main current which flows to the external circuit, there will be in each couple a circulating current, the path of which is diagrammatically indicated in Fig. 3. This circulating current will flow even when the external circuit is open, and its magnitude will vary with the drop of P.D. between the extreme ends of the sheathing of each couple, so that the greater the current in the external circuit, the smaller will be the circulating current in each couple.

In order to determine under a given set of conditions the effect of this circulating current, the following experiment was made: A strip of flattened 36 S.W.G. constantan wire had a piece of strip made from a 40 S.W.G. copper wire similarly flattened, soldered to its mid-point. The copper strip and one-half of the constantan strip were stretched tightly round a drum, and a heater was placed in a groove in the drum immediately beneath the soldered junction between the copper and constantan strips. The other half of the constantan strip was stretched round the drum over the copper strip, but with a thin piece of tissue paper separating the two, as seen in Fig. 4, this arrangement representing an ordinary end-to-end junction. A current was passed through the heater, and the resulting E.M.F. between the ends of the copper and constantan strips (1) and (2) was noted.

The paper insulation between the copper and constantan strips (1) and (3) was then removed, so that these two strips were in contact throughout their length, thus roughly representing a junction formed by copper plating one side of a constantan strip. With the same current through the heater, the E.M.F. between (1) and (2) was again noted, and was found to be 6·2 per cent. lower than in the first case. This result can only be regarded as approximate, since the contact

between the strips could not have been so perfect as if they had been formed by plating the copper upon the constantan.

The first practical application of the principles involved was tried for the measurement of small currents. A single junction constructed as described in Fig. 1, with a 0·002 in. diameter constantan wire having a copper sheath, was stretched between two supports, and arranged to pass through the centre of a small heating coil in the form of a solenoid, the junction lying inside the heating coil without being in electrical contact with it. This was found to be very satisfactory when employed for the measurement of small alternating currents after being calibrated on direct current.

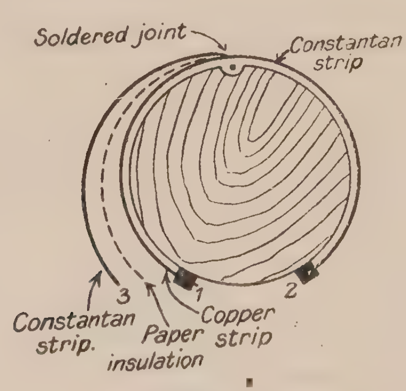


Fig. 4.

For various reasons, it appeared that for small junctions silver would be generally more satisfactory than copper for sheathing, and after frequent experiments this view was confirmed.

As it is desirable to avoid excess of metal in the silver sheath, which would result in conducting away heat more rapidly from the heated junction, beside requiring more heat to raise the larger mass of metal to a given temperature, rough trials were made, and it appeared that a suitable area for the silver sheathing was about 25 per cent. of that of the constantan core. Later, further tests were made, the results of which are shown in the curves in Fig. 5, from which it appears that the value originally arrived at was too low, and that 30 to 35 per cent. is a more suitable value.

In the test nine sets of junctions having different percentages of silver sheathing were connected respectively to

a moving coil ammeter, and were arranged equidistant from a straight wire heater through which a heating current was passed.

The method was now tried for constructing junctions of large cross-section which would be capable of dealing with heavier currents. For this purpose a piece of 14 S.W.G. constantan wire was silver plated for a portion of its length, so that the cross-section of the silver sheath was about 25 per

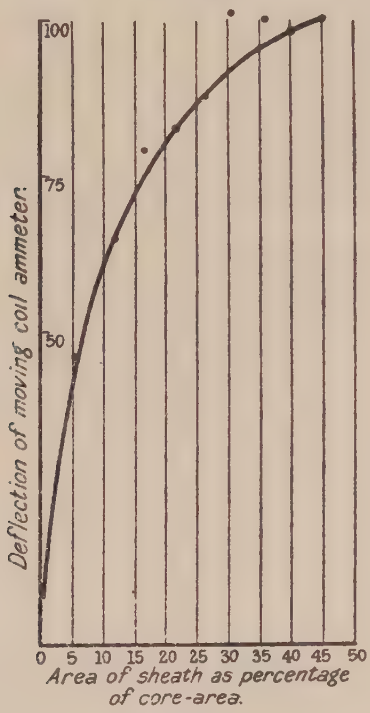


Fig. 5.

cent. of that of the constantan wire. The resistance of this junction was 0·017 ohm. On connecting the ends of the junction to the heavy brass terminals of a moving coil ammeter having a resistance of 0·01 ohm a current of 1 ampere was obtained when the junction was raised to a dull red heat by means of a spirit flame. On replacing the ammeter by a voltmeter of 500 ohms resistance, the instrument read 0·025 volt, with about the same temperature.

A junction made from a constantan strip, about 9/64 in. by 0·017 in., and silver plated for half its length, gave a rather better result, owing to the greater cooling surface obtained. After working this latter junction at red heat for some time, the silver sheathing was stripped back a short distance to see if the surface between the silver sheath and the constantan strip, where it was strongly heated, had undergone deterioration by oxidation. A slight trace of oxidation was observed for a distance of about 1/16 in. beyond the beginning of the sheathing, but beyond this point the surface was quite bright. This junction has been used repeatedly since, although slightly damaged by this stripping, without noticeable decrease in its effectiveness.

In order to decide what would be the most efficient distance to have between the hot and cold junctions, with a given size

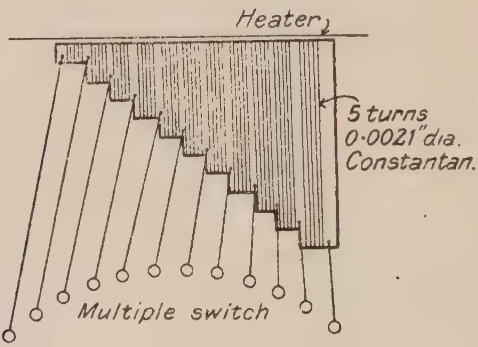


Fig 6.

of round conductor, having regard to the fact that the resistance must be as low as possible, while the difference of temperature between the junctions must be as great as possible, eleven sets, each of five junctions of 0·0021 in. diameter constantan, were made up, and mounted opposite a heating wire as shown in Fig. 6.

When a current of 0·08 ampere was maintained through the heater, E.M.F.s were generated by the respective groups of junctions as shown in Table A. Under these conditions it appears that with that size of wire 0·45 in. is the most suitable distance between the hot and cold junctions with air cooling.

TABLE A.

Distance between hot and cold junctions.	Voltage generated.
0·5 in.	0·001628 volt.
0·45 in.	0·0016 ,,
0·4 in.	0·001432 ,,
0·35 in.	0·001132 ,,
0·3 in.	0·000978 ,,
0·25 in.	0·000872 ,,
0·2 in.	0·000777 ,,
0·15 in.	0·000519 ,,
0·1 in.	0·000267 ,,
0·05 in.	0·000287 ,,

As the thermal conductivity of the uncoated portions of the junctions is very much lower than that of the coated portions, while the resistance is also higher, it is necessary, in order to obtain the highest efficiency, that the uncoated portion should be made as short as possible.

To determine the best relative lengths for coated and uncoated portions the arrangement shown in Fig. 7 was made.

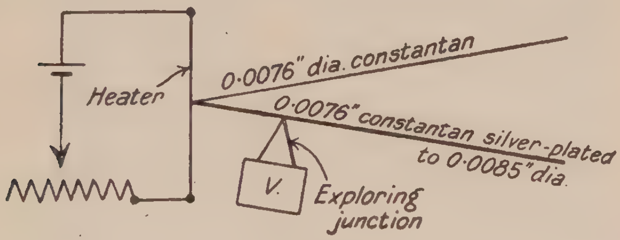


Fig. 7.

The small exploring junction was made from a 0·0016 in. constantan wire with a silver sheathing.

The results obtained are shown in Table B. :—

TABLE B.

Heating current.	Exploring junction current.	Distance of exploring junction from heater.	Ratio.
2 amp. Silver.......	2 micro-amp.	26 mm.	1·575
2 amp. Constantan	2 ,,	16·5 mm.	...
2 amp. Silver.......	6·2 ,,	17·5 mm.	1·59
2 amp. Constantan	6·2 ,,	11 mm.	...

This experiment shows that with a junction formed by silver-plating a 0·0076 in. diameter constantan wire to 0·0085 in.

diameter arranged horizontally in air, the temperature of any point on the uncoated wire will be the same as that of a point on the coated wire situated 1·58 times the distance from the heater.

In a similar manner the most efficient relative lengths to use under any other conditions may be determined.

In order to determine the most efficient distance between the heater and the junctions when these are not in actual

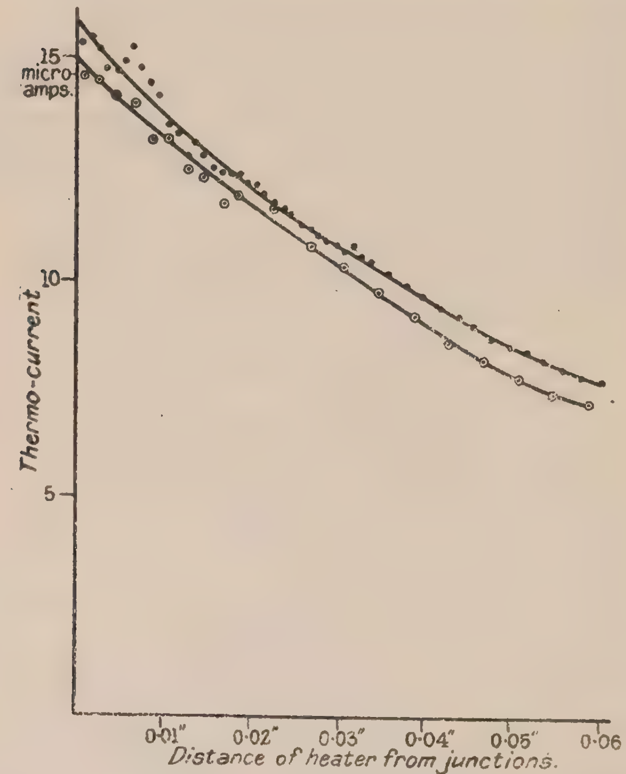


Fig. 8.

contact, an experiment was made with the heater mounted horizontally opposite a line of junctions constructed as already described, the heater being arranged with a micrometer to regulate the distance between it and the junctions. The results of the two tests are shown in Fig. 8. From this it appears that under those conditions the loss of efficiency

appears to be serious when the distance between the junctions and the heater is greater than 0·01 in.

For a line of junctions, the arrangement shown in Fig. 9 was found satisfactory. In this the core was of ebonite, hollowed out to facilitate cooling, and the conductor consisted of silver-plated constantan strip about 0·008 in. by 0·00075 in. Sixty junctions had a resistance of 85 ohms.

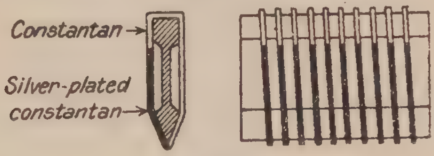


FIG. 9.

Mica was found more convenient than ebonite for many purposes, but it was soon apparent that the presence of any insulating material near the hot junctions materially lowered their temperature, besides rendering their action very much

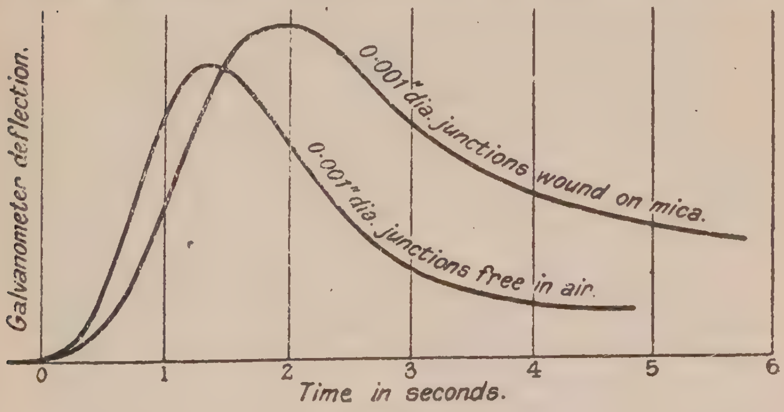


FIG. 10

more sluggish. The effect of this is well seen in the photographic records in Fig. 10, for which I am indebted to Prof. A. V. Hill, F.R.S., and Mr. W. Hartree, of the Physiological Laboratory, Cambridge.

The arrangement for mounting the junctions shown in Fig. 11 was therefore adopted, whereby the proximity of insulating material to the junctions is avoided. This arrange-

ment is the same as that adopted by Prof. Hill in some of his work, in which he used junctions constructed in the usual manner (" Journal of Physiology," Vol. XLIII., No. 6, February, 1912).

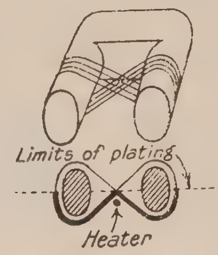


Fig. 11.

In order to deal with higher temperatures than could safely be used with silver and constantan, junctions were constructed by nickel-plating wires of nickel-chrome alloy specific resistance 575 ohms per circular mill foot, 0·032 in. diameter before plating and 0·0445 in. after plating. These junctions gave approximately 0·016 volt when the difference of temperature between the hot and cold junctions was about 900°F.

Owing to the ease with which junctions may be constructed in series by this method, and in very varied forms, a large number of arrangements has been devised, each with a special end in view.

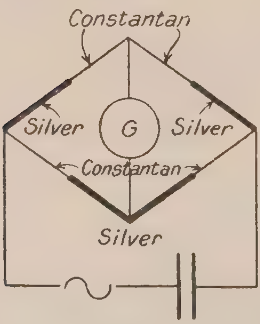


Fig. 12.

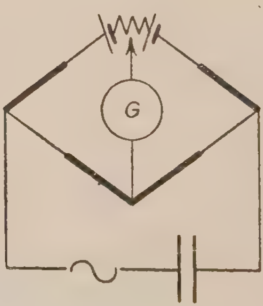


Fig. 13.

One type is in the form of a Wheatstone bridge arrangement (Fig. 12), the arms of which are balanced for resistance, but have thermo-junctions so arranged that the thermo-E.M.F.s cancel in the external or supply circuit, but assist one another

in sending current through the measuring instrument in the branch circuit.* This arrangement utilises the high resistance portion of each junction as a heater, and is particularly useful in measuring high frequency alternating currents, since the inductance and resistance of the arms may be properly balanced so that none of the high-frequency current flows through the instrument, and the arrangement can be accurately calibrated on continuous current owing to the fact that the differential action of the thermo-junctions tends to maintain the sum of the thermo-E.M.F.s constant even though the alternating current may not divide between the two paths in precisely the same ratio as the continuous calibrating current. The error would be less than $1\frac{1}{4}$ per cent. even with 25 per cent. more current in one branch of the bridge than in the other.

For dealing with large currents it is convenient to have a series of thermo-junctions in each arm of the bridge. These may be constructed as shown in Fig. 14; the junctions here are made with much larger cross-sections and radiating sur-

FIG. 14

faces at the cold junctions, so that the heat generated by the passage of the current results in a higher temperature at the constricted sections, and thus a series of preponderating E.M.F.s are set up along the strips in one direction.

One bridge constructed in this manner had a resistance of 0·37 ohm, and with an alternating current of 6·75 amperes through the bridge the thermo-E.M.F. generated was 0·0192 volt.

The above arrangement may be made more sensitive by inserting two equal sources of E.M.F. as shown in Fig. 13, and readjusting the balance of the bridge to compensate for the thermo-E.M.F.s set up by the circulating current generated by these sources of E.M.F. round the bridge circuit.

In certain respects these Wheatstone bridge arrangements are similar to the bolometric arrangement used in 1890 by

* Since writing this Paper, we find that a similar arrangement, using the ordinary type of couples, has been described in British Patent No. 5413 of 1909.

H. Rubens and R. Ritter (" Weid. Annalen," Vol. XL., p. 56), while if the thermo-E.M.F.s are produced by the absorption of radiant heat instead of by a heating current they resemble Langley's bolometer. The chief difference from those arrangements is that the galvanometer is deflected by the thermo-E.M.F.s set up in each arm of the bridge instead of by a change in resistance of one arm.

Another useful arrangement consists of two lines of junctions connected in opposition and arranged close together as shown in Fig. 15. If radiant heat be arranged to fall in a line 1 mm. in width symmetrically about the axis A-B, the thermo-E.M.F.s generated in the two halves will be equal and opposite, but a movement of the heat line of ½ mm. to either side will

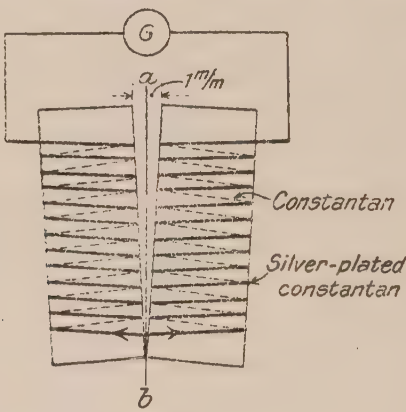


Fig. 15.

cause it to cover entirely one set of junctions or the other, resulting in a deflection of the galvanometer from one side of zero to the other side of zero. Since the number of junctions in each line may be made large by these methods, the deflection of the galvanometer G may be made substantially proportional to the movement of the band of radiant heat.

The arrangement, therefore, comprises a useful means of magnifying small movements. The band of radiant heat may be the image formed by the mirror of another reflecting galvanometer, hence it is possible to cause a deflection produced by this galvanometer to produce a considerably larger deflection in the galvanometer G connected in the thermo-electric circuit.

For such work as spectrum analysis the method enables lines of junctions in series to be constructed with ease and

accuracy, which present a very small width to the radiation to be analysed, and should prove valuable for such purposes.

It is necessary to guard against the presence of insulating material in proximity to the working line of junctions as heat is absorbed by such material, and acts upon the junctions so that the full benefits are not obtained which would arise from the narrowness of the line of junctions.

For work where couples of exceedingly small mass are required, the methods offer great possibilities, since the only limitation to the fineness of the junctions is that imposed by mechanical questions of producing the wire. In this way it has been found possible to construct junctions with a diameter of less than 0·0002 in., and there seems no reason why this linit should not be exceeded.

Such junctions, owing to their very small mass, respond with great rapidity to changes of temperature.

For some purposes it is convenient to have a very large number of junctions connected in series, and occupying the smallest possible space. This may be readily done by forming lines of junctions on slips of mica, and assembling them with mica or paper between. In this way a pile has been constructed having 4,000 couples in series with a resistance of about 8,000 ohms, the hot junctions presenting an area of $1\frac{1}{2}$ in. by $\frac{5}{8}$ in.

ABSTRACT.

The method, which was devised to overcome the difficulty of making satisfactory soldered joints between the elements of thermopiles having a large number of closely packed junctions, consists in using a continuous wire of one of the elements and coating those parts of it which have to form the other element with an electrolytic deposit of another metal. If the conductivity of the latter is considerably greater than of the former, and a fairly thick sheath is deposited, a thermo-couple is produced which is not appreciably impaired in efficiency by the short-circuiting effect of the core. Constantan wires coated with either copper or silver sheaths were found to be suitable for most purposes.

DISCUSSION.

Mr. R. S. WHIPPLE said there were many cases in which the device would be of the greatest service to physicists, in the measurement of radiation, for instance. He might mention that Prof. Hill, of Cambridge, has recently been measuring the rise of temperature of nerves in action by means of these couples.

Mr. F. E. SMITH said that he had had to make a number of thermopiles at one time and another, and he fully appreciated the advantage of Mr. Wilson's method. It would be a great labour-saving device.

Mr. C. R. DARLING asked if the method would enable a radiation pyrometer similar to the Féry type to be made for the measurement of low surface temperatures.

Dr. J. S. ANDERSON emphasised the utility of the device for spectro-radiation work, as the total breadth could be made very small and a large number of junctions per centimetre employed. He had had the opportunity of using some of Mr. Wilson's couples, and was very pleased with them.

Dr. RAYNER asked what difference of temperature could be detected. He thought the measurements should be given in the metric system to render the Paper comprehensible to foreigners.

Dr. VINCENT asked if Wollaston wire had been tried. Had the winding to be done by hand, or could it be done on a lathe?

Prof. FORTESCUE suggested that the arrangement of Fig. 14 could be adapted to the measurement of quite high-frequency currents.

Dr. L. HOPWOOD asked if the Authors were certain of the effect which was utilised in the arrangement of Fig. 14. Benedicks had shown that if there was a steep temperature gradient, as would be the case in this arrangement, a reversed Thomson effect, comparable in magnitude to the Seebeck effect, was obtained. It would be interesting, therefore, to try using only one metal instead of two.

Mr. WILSON said there would be no difficulty in measuring low surface-temperatures with an instrument employing these junctions. As regards measurable temperature difference, he had not gone beyond about 1/1,000°C. He had not tried Wollaston wire, but he found that a patent had recently been applied for using such wire and dissolving off the sheath from one "element." Many of the couples shown were wound on lathes. In other cases the grooves were lathe-cut and the wire put on by hand, which was almost as quick. He had not made experiments with one metal with the arrangement of Fig. 14 as suggested by Dr. Hopwood, but would do so at the first opportunity. His results, however, appeared to be due to ordinary thermo-electric effects.

XXX. *On the Use of "Vacuum Arcs" for Interferometry.* By

J. Guild, *A.R.C.Sc., D.I.C., F.R.A.S.* (From the National Physical Laboratory.)

Received June 5, 1920.

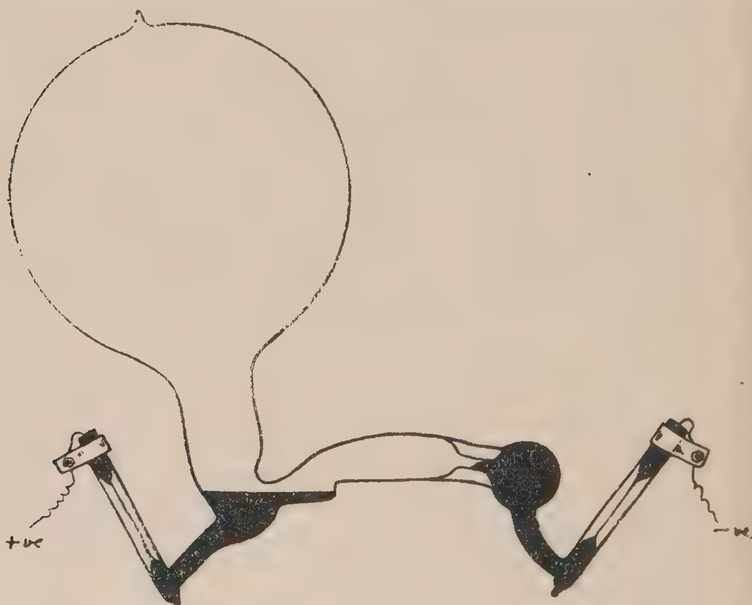
In the use of interferometers of various types it is frequently necessary to obtain interference fringes of considerable path difference—for example, in testing the plane parallelism of thick slabs of optical glass. This necessitates the use of light which is monochromatic to a very high degree; and only radiations which give very narrow spectrum lines are suitable for such work. On account of the many convenient forms of mercury vapour lamp which have been available in recent years, the mercury lines, particularly the green line at 5461 A.U., are almost universally used for general interferometer work. It is found, however, that such lamps vary greatly in their suitability if the path difference is appreciable. The short quartz lamps of high intrinsic brilliancy are practically useless for any but quite thin films; whereas the long lamps, with an arc length of about 4 ft., designed for the lighting of workshops, &c., give satisfactory fringes for quite considerable path differences.

The reason for this is, of course, obvious. The short arc consumes approximately as much energy as the long one, and since this is dissipated in much less space, the rise in temperature of the lamp is correspondingly greater. The vapour pressure of the mercury is, therefore, high in the short lamps; and the spectrum lines are in consequence broadened considerably. This results in a serious loss of fringe visibility for large path differences.

For many purposes the length of the long tube arc is not particularly inconvenient. Moreover, it is sometimes an advantage, as by the aid of mirrors and lenses one lamp may be made to illuminate several interference apparatus distributed around a room. For other purposes, however, its size and non-portability are objectionable, as it may be difficult to arrange it in the proper relationship to other parts of the apparatus. Not only so, but it is frequently desirable to have a source of greater brilliancy, but giving at the same time the narrow lines of the long arc. This can be accomplished quite simply. The vapour pressure in an enclosure is determined by the temperature of the coolest

part. Even if the rate of evaporation in the hot part is very rapid, no great excess of pressure can exist unless there is a narrow constriction between the hot and cold regions.

The pressure in the short arc can, therefore, be considerably reduced by attaching a condensing chamber to the lamp. This has been done in a lamp designed by the author, and made by the Westinghouse Cooper Hewitt Co., Ltd. It consists of their ordinary laboratory quartz lamp, with the addition of a large bulb above the positive electrode which serves as a condensing chamber to keep the pressure low.



The diameter of the bulb is about 5 inches and the general proportions of the lamp are shown in the sketch.

The performance of this lamp is very satisfactory. When taking a current of 2·5 to 3 amperes, it produced visible fringes on an interferometer with as great a path difference as the four-foot arc, and the illumination of the field was very much brighter. With a current of 5 amperes, which is as much as this type of lamp should be run at for regular use, there is a very slight deterioration in the fringe visibility with long paths, but this is not sufficient to matter in practice.

It may be of interest to give some comparative figures of the performance of four mercury arcs of different types. The figures given are the maximum path differences, in air, for which fringes were observable with a particular interferometer arrangement.*

Arons-Lummer Water-Cooled Quartz Arc, in which the arc is in the form of a horse-shoe, enclosed in an outer cover, also of quartz, which is water-cooled.	4 cm.
Westinghouse Cooper Hewitt Quartz Lamp, type Y_1 ..	2 cm.†
Similar Lamp with Condenser, as described above	10 cm.
Westinghouse Cooper Hewitt Lamp, type K (*4-ft. arc*) .	10 cm.

It is clear from these figures that the first two lamps, representative of the ordinary patterns of short laboratory arc, are of very limited utility in general interference work as compared with either the third or fourth. Type three, by combining the high intrinsic brightness of the short arc‡ with the greater spectral homogeneity of the long arc, will be found of considerable utility by many workers.

It has not been possible to try this expedient with the Cadmium arc. Unfortunately the Cadmium lines, from their simple structure the most suitable of all for interference work, are not readily obtained of satisfactory brightness and homogeneity at the same time. For purposes where extreme homogeneity is not required, *e.g.*, polarimetry, the enclosed Cadmium vacuum arc of Dr. Sands§ gives a beautifully intense Cadmium spectrum; but owing to the high temperature which the lamp attains the lines are ruined for interference work, except with thin films.

There would probably be a difficulty in employing a condensing bulb to lower the pressure in a Cadmium arc, as the condensed metal might stick to the bulb instead of falling back into the arc. If this difficulty could be overcome, say, by choosing the bulb of such size that the Cadmium vapour was liquefied but not solidified, the use of these lines for interferometry might be greatly facilitated.

* The visibility depends on several factors besides the homogeneity of the light; the figures are, therefore, comparative only.

† After running for some minutes so as to obtain a steady state.

‡ The intrinsic brilliancy of the quartz arc with condenser is distinctly less than that of the ordinary pattern for the same power consumption, but is much greater than that of the long arc.

§ Proc. Phys. Soc., Vol. XXVIII., p. 94.

ABSTRACT.

The Paper discusses the relative merits of short and long mercury arcs for this work, and points out that the defect of the former is due to the broadening of the spectrum lines consequent on the high vapour pressure within the lamp. It is shown that by attaching a condensing bulb to the lamp, so as to prevent excessive rise of vapour pressure, the short lamp can be made practically as good as the long one as regards sharpness of lines, while still being of much greater intrinsic brightness.

XXXI. *The Maintenance of a Vibrating System by Means of a Triode Valve.* *By* S. BUTTERWORTH, *M.Sc.* (*From the National Physical Laboratory.*)

RECEIVED MAY 27, 1920.

1. THE triode valve has been applied recently by Eccles* for the purpose of maintaining the vibrations of a tuning fork. The circuit employed is shown in Fig. 1. V is the triode valve, whose filament F is heated by the battery FB. The anode A is connected through the anode battery AB, and the coil AC to one terminal of the filament. The grid G is connected to the

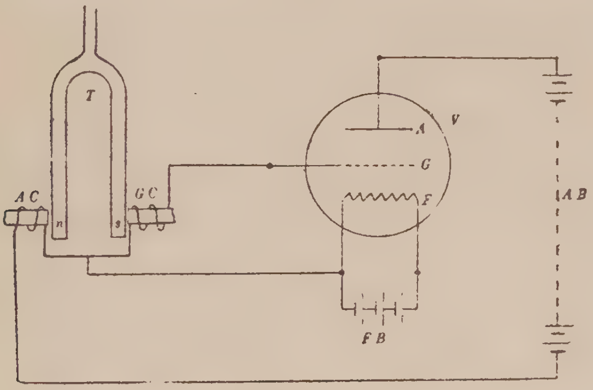


FIG. 1.

filament through the coil GC. The tuning fork T is permanently magnetised by an auxiliary magnet, on whose pole-pieces the coils AC and GC are wound, the poles of the fork being indicated by n and s.

The action is as follows: Suppose the fork to have been set in vibration, and consider the moment when the two prongs are moving away from their respective coils. The motion of the pole s will induce an E.M.F. in the coil GC, which will raise the potential of the grid, and so increase the current flowing into the valve at the anode. This current leaving by the filament completes its circuit via the coil AC, round which it passes in such a direction as to repel the pole n, and thus assist its motion.

* Eccles, Proc. Phys. Soc., Vol. XXXI., p. 269, 1919.

The same argument applies to any phase of the motion so that the vibrations are maintained.

The motion is only assisted if the relative directions of winding in the two coils are as in the figure. If the ends of the coils be disconnected from the anode battery and the grid respectively, and the open ends connected to a source of alternating current, the force exerted by the current flowing through the grid coil would oppose that due to the current in the anode coil if the winding is correct.

2. It is also necessary that the energy supplied by the valve shall be sufficient to balance the energy dissipated by the motion of the fork. If the fork be treated as a system of one degree of freedom, this condition may readily be determined.

Let A_1 be the mechanical force acting on the fork when unit current flows in the anode coil. Then, if a current I_a leaves the valve by the anode, the equation of motion of the fork is

$$(\alpha D^2+\beta D+\gamma)y=A_1 I_a \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

in which α, β, γ are the usual mechanical constants, y is the instantaneous deflection and $D=d/dt$. If A_2 is the force per unit current flowing in the grid coil, the voltage induced in the grid coil raises the potential of the grid by an amount

$$v=A_2 Dy \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (2)$$

A_1 and A_2 having the same sign when the coils have the same direction of winding, so that the extra anode current is

$$i_a=-gv=-gA_2 Dy \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (3)$$

in which g is the slope of the grid voltage — anode current characteristic of the valve at its working point, g being of the dimensions of a conductance and positive in sign.*

Since the normal anode current merely produces a steady deflection, we may, for the purpose of determining the motion, identify i_a with I_a, so that (1) becomes

$$\{\alpha D^2+(\beta+A_1 A_2 g)D+\gamma\}\, y=0. \quad . \quad . \quad . \quad . \quad . \quad (4)$$

The oscillations will increase, remain steady or decay, according as $\beta+A_1A_2g$ is negative, zero or positive.

Thus, for the maintenance of oscillations, A_1 and A_2 must be of opposite signs, and

$$g>-\beta/A_1A_2=g_0(\text{say}). \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (5)$$

* The effect of variations of anode voltage upon anode current is discussed in section 18.

The conclusion in regard to sign is identical with that arrived at in section I.

3. The quantity β/A_1A_2 is of the dimensions of a conductance, and it is important to form some notion of its magnitude. Suppose the anode and grid coils to have a mean radius of 1 cm., and a square channel section of 1 cm. side; and let them be wound with 10,000 turns of fine wire. The strength of the poles on the fork will be taken as 200 C.G.S. units. Then, assuming the turns concentrated in a single ring of 1 cm. radius, with the pole as centre, the force urging the pole when unit current flows is 4π megadynes. This gives the order of A_1 or A_2 if we may treat the fork as if its inertia were concentrated at the poles. Now, Rayleigh* shows that this may be done for the fundamental period, and that the equivalent inertia of one prong is then one-quarter its whole mass, the remainder of the prong acting as a spring devoid of inertia. This gives $\alpha \doteq \frac{1}{2}$ (mass of one prong), since both prongs are vibrating. With a fork of reasonable dimensions, the mass of one prong may be taken as of the order 20 grams, and then $\alpha=10$ grams.

β can be found from the rate of decay of the free oscillations and the value of α. Thus, suppose the oscillations to fall to $1/e$th of their initial value in one second; then $\beta=2\alpha=20$.

Hence, taking $A_1=A_2=10^7$,

$$A_1A_2/\beta=5\times10^{12}\text{ C.G.S. units}=5{,}000\text{ ohms},$$

since it is of the dimensions of a resistance.

The condition for maintenance of the above system is, therefore, that 1 volt on the grid should develop an anode current of at least 200 microamperes.

A coil of the above dimensions and turns wound with No. 46 wire would have a direct current resistance of about 4,000 ohms, and its inductance with an air core would be about 4 henries.

For other turns and the same overall dimensions, A_1, A_2, the resistance and the inductance all increase as the square of the turns.

4. It has been suggested† that a condenser placed in parallel across the anode or grid coil would render the oscillations easier to maintain, and this has proved of value in certain cases. That a condenser might improve matters is to be ex-

* "Theory of Sound," Vol. I., p. 289.
† Proc. Phys. Soc. (*loc. cit*). Discussion.

pected from the principles of resonance, although when a considerable current is flowing in the condenser-coil circuit there is an additional energy loss due to the resistance of the coil. Again, the optimum capacity will not necessarily be that which will make the free frequency of the electrical circuit equal to that of the fork, as the reactive effect due to the motion of the fork is by no means negligible.

5. In the following theory use is made of a principle of electrical equivalence developed by the author* in an earlier Paper, in which it was shown that a dynamical system kept in motion by the magnetic action of a current flowing in a coil could be replaced by an electrical system in series with the coil, and of such a nature as to produce the same back E.M.F. as that produced by the motion of the dynamical system.

For the case of a system of one degree of freedom, the equivalent electrical system consists of a parallel combination of inductance, capacity and conductance. In the present problem the only difference is that two coils are acting instead of one. The necessary modifications on the earlier theory are easily made.

6. If the two coils carry currents, I_1 and I_2, they act on the dynamical system with forces A_1I_1 and A_2I_2. The equation of motion is

$$(aD^2+\beta D+\gamma)y=A_1I_1+A_2I_2 \quad . \quad . \quad . \quad . \quad . \quad (6)$$

and the back E.M.F. on the respective coils are

$$E_1=A_1Dy, \quad E_2=A_2Dy \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (7)$$

so that, eliminating y by means of (6),

$$E_1=I_1/\xi_1+I_2/\xi_{12}, \; E_2=I_1/\xi_{12}+I_2/\xi_2 \quad . \quad . \quad . \quad (8)$$

in which

$$A_1^2\xi_1=A_1A_2\xi_{12}=A_2^2\xi_2=aD+\beta+\gamma/D \quad . \quad . \quad . \quad (9)$$

Equations identical with (8) will be obtained if, instead of the dynamical system, we place in series with the respective coils inductances (without resistance) of values A_1^2/γ, A_2^2/γ, perfectly coupled with each other, and shunt the former by means of a leaky condenser of capacity a/A_1^2 and leakage conductance β/A_1^2. The leaky condenser may be transferred if desired to the second coil, the values of capacity and leakance being then a/A_2^2 and β/A_2^2. This follows from the properties of a perfectly coupled transformer.

* Proc. Phys. Soc., Vol. XXVII., p. 410, 1915.

7. Applying this principle to the case of the maintained fork, the system may now be taken as purely electrical, the circuits being shown in Fig. 2. l_1, l_2 are the equivalent inductances of values A_1^2/γ, A_2^2/γ perfectly coupled. c_1 is the equivalent condenser α/A_1^2, and s_1 its leakage conductance β/A_1^2. L_1 is the inductance and R_1 the resistance of the anode coil, L_2 R_2 the inductance and resistance of the grid coil. The condensers whose effects are to be investigated are C_1 and C_2.

The action of the equivalent leaky condenser may be taken into account by first obtaining the equations which hold when the condenser is absent, and then replacing the resistance operators l_1D, l_2D and the mutual operator $\sqrt{l_1 l_2}D$ by $1/\xi_1$, $1/\xi_2$, $1/\xi_{12}$, the values of ξ_1, ξ_2, ξ_{12} being given by (9).

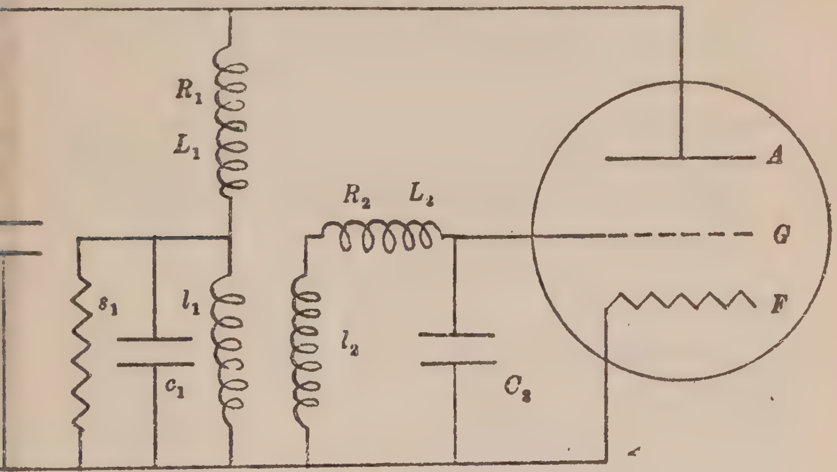


FIG. 2.

Let i_1, i_2 be the clockwise currents* circulating in the anode and grid circuits respectively. The grid voltage is i_2/C_2D, and the resulting anode current leaving the valve is $-gi_2/C_2D$. This current flows back to the filament via C_1, thus impressing an E.M.F. on the anode circuit of amount $-gi_2/C_1C_2D^2$. Hence, if X_1, X_2 are the resistance operators of the anode and grid circuits, and m is the mutual operator,

$$\left.\begin{aligned} X_1 i_1 &= (m - g/C_1C_2D^2)i_2 \\ X_2 i_2 &= m i_1 \end{aligned}\right\} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (10)$$

* The currents and voltages referred to are the deviations of the currents and voltages from the normal values when oscillations are excited.

from which

$$(X_1X_2/m - m + g/C_1C_2D^2)\{i_1, i_2\} = 0 \quad . \quad . \quad . \quad . \quad (11)$$

Using the explicit values of X_1, X_2, m, viz.,

$$X_1 = R_1 + L_1D + 1/C_1D + 1/\xi_1, \; X_2 = R_2 + L_2D + 1/C_2D + 1/\xi_2, \; m = 1/\xi_{12},$$

(11) becomes, with the help of (9),

$$\{(\alpha D^2 + \beta D + \gamma)(C_1L_1D^2 + C_1R_1D + 1)(C_2L_2D^2 + C_2R_2D + 1) + A_1^2C_1D^2(C_2L_2D^2 + C_2R_2D + 1) + A_2^2C_2D^2(C_1L_1D^2 + C_1R_1D + 1) + A_1A_2gD\}\{i_1, i_2\} = 0. \quad . \quad . \quad . \quad (12)$$

The three possible oscillation frequencies indicated by (12) are obtained by putting $D^2 = -\omega^2$ in the operator, thus giving a complex in ω, of which both the real and imaginary parts must be zero. The real part is a cubic in ω^2, whose three roots determine the frequencies $(\omega/2\pi)$. The imaginary part gives the values of g, which will just maintain these frequencies.

8. The solution of (12) in the general case would be difficult, but the case of similar anode and grid circuits is amenable to treatment if the dampings are not too large. Putting $C_1 = C_2 = C$, $L_1 = L_2 = L$, &c., and writing $-\omega^2$ for D^2, the equation to determine the frequencies is

$$(\gamma - \alpha\omega^2)(1 - CL\omega^2)^2 - 2A^2C\omega^2(1 - CL\omega^2)(1 + \beta R/A^2) - C^2R^2\omega^2(\gamma - \alpha\omega^2) = 0 \quad . \quad . \quad . \quad (13)$$

and the equation to be satisfied by g (the conditional equation) is

$$\beta(1 - CL\omega^2)^2 + 2CR\{(\gamma - \alpha\omega^2)(1 - CL\omega^2) - A^2C\omega^2(1 + \beta R/2A^2)\} \pm A^2g = 0 \quad . \quad . \quad . \quad (14)$$

The positive sign is used when the coils are similarly wound and the negative sign when oppositely wound.

Putting further,

$$\gamma = \alpha\omega_1^2, \; 1/CL = \omega_2^2, \; R/L = \mu\omega_1, \; \beta/\alpha = \nu\omega_1 2(A^2/L\alpha)(1 + \beta R/A^2) = \sigma^2\omega_1^2,$$

(13) and (14) become

$$(\omega_1^2 - \omega^2)(\omega_2^2 - \omega^2)^2 - \sigma^2\omega^2\omega_1^2(\omega_2^2 - \omega^2) - \mu^2\omega^2\omega_1^2(\omega_1^2 - \omega^2) = 0 \quad . \quad . \quad . \quad (15)$$

$$\nu\{(\omega_2^2 - \omega^2)^2 + \mu^2\omega^2\omega_1^2\} + \mu\{2\omega_1^2 - \omega^2)(\omega_2^2 - \omega^2) - \sigma^2\omega^2\omega_1^2\} \pm A^2g\omega_2^4/\omega_1\alpha = 0 \quad . \quad . \quad . \quad (16)$$

9. In the case when the free frequency of the circuits is equal to that of the fork (that is, $\omega_2=\omega_1$), the frequency equation immediately factorises, giving

$$\omega^2/\omega_1^2=1,\ 1+\tfrac{1}{2}(\sigma^2+\mu^2)\pm[\{1+\tfrac{1}{2}(\sigma^2+\mu^2)\}^2-1]^{\frac{1}{2}}.$$

Thus, the three possible oscillation frequencies are respectively below, equal to, and above the free frequency of the fork. If, as is often the case, σ and μ are small compared with unity,

$$\omega/\omega_1\doteqdot 1,\ 1\pm\tfrac{1}{2}(\sigma^2+\mu^2)^{\frac{1}{2}}.$$

For the root $\omega=\omega_1$, the conditional equation reduces to

$$\mu(\nu\mu-\sigma^2)\pm A^2g/\omega_1 a=0.$$

Now, $\sigma^2>\nu\mu$, so that for the above condition to be possible the coils must be *similarly* wound. Inserting the values of σ, μ, ν, and using the positive sign,

$$g=2R(1+\beta R/2A^2)/L^2\omega^2 \quad \ldots\ldots \quad (17)$$

For the other roots, the conditional equation reduces to

$$(\nu+\mu)(\sigma^2+2\mu^2)\omega^2\pm A^2g\omega_1/a=0.$$

The negative sign must be used, so that these oscillations can only occur when the coils are *oppositely* wound. The *lower* of the two frequencies requires the smaller value of g, so that this will be the actual frequency of the oscillation. When σ and μ are small, $\omega\doteqdot\omega_1$, and the condition is then

$$g=(\nu+\mu)(\sigma^2+2\mu^2)a\omega_1/A^2. \quad \ldots\ldots \quad (18)$$

In the typical example of section 3, $\sigma^2=0{\cdot}00025$, $\mu^2=0{\cdot}025$, $\nu^2=10^{-7}$, a free fork frequency of 1,000 cycles per second being assumed. With these values, we find for similarly wound coils (equation 17) $1/g=50{,}000$ ohms; for oppositely wound coils (equation 18), $1/g=200$ ohms. The former condition corresponds to 20 microamperes per volt, and the latter to 5 milliamperes per volt. The oscillation with similarly wound coils is, therefore, by far the easiest to maintain. The value of g for the oppositely wound coils will, however, rapidly diminish with increase of L/R, because of the factor μ^3. Also, as will be shown in the next section, a small variation of ω_2 (by capacity adjustment) will cause a gain in ease of maintenance.

10. If the frequency of the electrical circuits is varied over a short range in the neighbourhood of $\omega_1/2\pi$, we may write in (15) and (16)

$$\omega_2^2-\omega^2=x\omega_1^2,\ \omega_1^2-\omega^2=y\omega_1^2;$$

and in the case σ^2, μ^2 small, they reduce to

$$x(xy-\sigma^2)-\mu^2y=0$$

$$\nu(x^2+\mu^2)+\mu(2xy-\sigma^2)\pm A^2g/\omega_1a=0\ ;$$

or, expressing g and y in terms of x

$$y=\sigma^2x/(x^2-\mu^2) \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (19)$$

$$\pm g=-(a\omega_1/A^2)(x^2+\mu^2)\{\nu x^2+\mu(\sigma^2-\mu\nu)\}/(x^2-\mu^2). \quad . \quad (20)$$

Since $\sigma^2>\mu\nu$, the oscillations which occur when the coils are similarly wound are such that $x^2>\mu^2$. In this case g is least when $x=0$, so that the value of g given in (17) is the minimum g for similarly wound coils.

The oscillations which occur when the coils are oppositely wound are such that $x^2>\mu^2$. A minimum value occurs for g when

$$x^2-\mu^2=\mu\sigma\sqrt{2\mu/\nu}, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (21)$$

and then

$$g=(a\omega_1/A^2)(\mu\sqrt{2\nu}+\sigma\sqrt{\mu})^2, \quad . \quad . \quad . \quad (22)$$

while

$$y=x\sigma\sqrt{\nu/2\mu^3}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (23)$$

In the typical case at minimum g

$$x\doteqdot 0{\cdot}32,\ y\doteqdot 0{\cdot}001,\ 1/g\doteqdot 15{,}000 \text{ ohms.}$$

11. The value of x in the typical case is thus hardly within the approximation assumed in obtaining (20); y is, however, small.

If we do not assume x small, equation (20) must be modified by replacing g in that equation by $g(1+x)^2$, y being still supposed small. We have thus really found when $g(1+x)^2$ is a minimum. The true minimum g will occur for a larger x when x is positive, and the value of g will be less than that calculated above. When x is negative, the minimum will occur for a smaller value of x, and the value of g will be larger than that for the positive x. Thus, of the two possible minima, the best is that for which $\omega_2>\omega_1$.

12. The full behaviour of the system in the neighbourhood of $\omega_2=\omega_1$ may be exhibited graphically for given values of σ, μ, ν, $a\omega_1/A^2$.

From equation (19) y is calculated for a suitable series of values of x, and $-y$ is plotted against $x-y$. The ordinates then give $(\omega^2-\omega_1{}^2)/\omega_1{}^2$, and the abscissæ $(\omega_2{}^2-\omega_1{}^2)/\omega_1{}^2$. The graph, therefore, gives the variation of the oscillation fre-

quency from the free frequency of the fork as the free electrical frequency is varied by capacity adjustment.

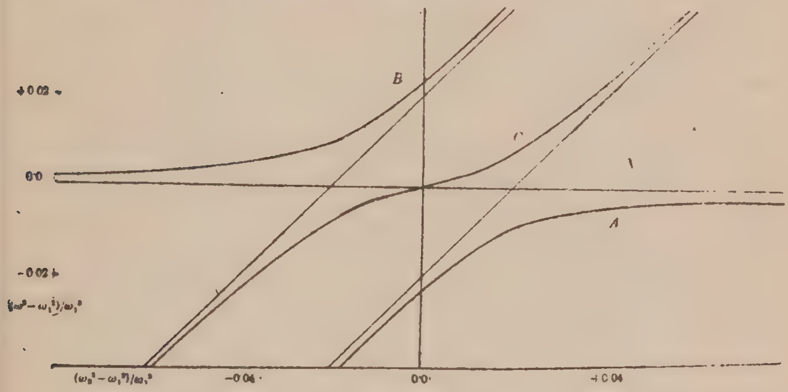


FIG. 3.

Similarly, from the (modified) equation (20), g is calculated and plotted against $x-y$. This graph shows the condition for maintenance of the oscillations given in the former graph.

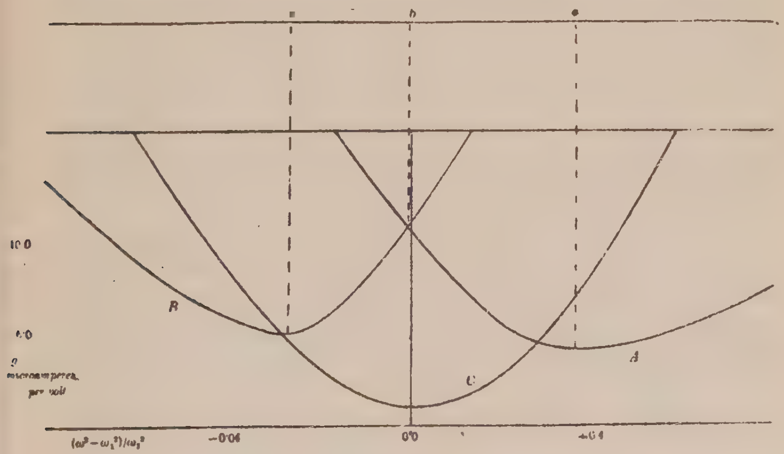


FIG. 3A.

$\omega_1/2\pi$=free frequency of fork. $\omega_2/2\pi$=free frequency of circuits. $\omega/2\pi$ =frequency of maintained oscillations.

The case used for Figs. 3 and 3A is

$$\sigma=0{\cdot}012,\ \mu=0{\cdot}02,\ \nu=0{\cdot}002,\ A^2/a\omega_1=2 \text{ ohms},$$

which corresponds, for example, to

$$\omega_1=5{,}000,\quad R=1{,}250 \text{ ohms},\quad L=12\cdot5 \text{ henry},\quad \beta/\alpha=10,\quad \alpha/A^2=100 \text{ microfarads}.$$

In Fig. 3 the possible oscillations when the coils are oppositely wound are given by the curves A and B. The curve C is for the single oscillation occurring when the coils are similarly wound. In Fig. 3A the values required for the three oscillations A, B, C are given by the curves with corresponding letters.

To trace what occurs as the capacity increases we must proceed from right to left along the horizontal axis. With opposite windings and capacities appreciably lower than the resonating capacity, it is the oscillation A which always occurs. This follows from the smaller value of g required to maintain A. Up to the point a the ease of maintenance for both A and B increases. From a to b, A becomes more difficult to maintain, while B still increases in ease of maintenance until at b the ease of maintenance of either oscillation is the same. A slight increase of capacity at the point b will make B easier to maintain than A, so that there will be a sudden increase in frequency from that corresponding to A to that corresponding to B. If the frequency difference is suitable, this will be accompanied by beats as A dies down, while B builds up. Beyond the point b it is the oscillation B only which is present, its ease of maintenance being a maximum at the point c.

As regards change of pitch with increase of capacity, there is a slow fall in frequency from the free fork frequency, this fall increasing in rapidity as the point of instability is approached. The sudden rise in pitch at the point B is followed by a slow fall towards the free fork frequency.

With similar windings (following the oscillation C), the frequency always remains near that of the electric circuits being lower when $\omega_2>\omega_1$ and higher when $\omega_1>\omega_2$. The oscillation is easiest to maintain at the resonating frequency, and its variation with respect to the frequency of the fork is least at this point.

13. When only one condenser is used, equation (12) reduces to

$$\{(\alpha D^2+\beta D+\gamma)(CLD^2+CRD+1)+A_1^2CD^2+A_1A_2gD\}\{i_1,\ i_2\}=0, \quad . \quad . \quad . \quad . \quad (24)$$

in which A_1 refers to that coil across which the condenser is placed.

Putting $D^2=-\omega^2$, and using the notation of equation (15), the frequency equation is

$$(\omega_1^2-\omega^2)(\omega_2^2-\omega^2)=\tfrac{1}{2}\sigma^2\omega_1^2\omega^2. \quad . \quad . \quad . \quad . \quad . \quad . \quad (25)$$

The two possible oscillation frequencies, therefore, lie outside the range $\omega_1 \backsim \omega_2$, one being above and the other below this range. Since usually σ^2 is small, the roots (Ω_1, Ω_2) of (25) do not differ much from ω_1 and ω_2, their approximate values being given by

$$(\Omega_1-\omega_1)/\omega_1=(\omega_2-\Omega_2)/\omega_2=\sigma^2\omega_1^2/4(\omega_1^2-\omega_2^2), \quad . \quad (26)$$

when ω_1 and ω_2 are not nearly equal, and by

$$\Omega_1-\omega_1=\omega_2-\Omega_2=\sigma\omega_1/2\sqrt{2} \quad . \quad . \quad . \quad . \quad (26\text{A})$$

when $\omega_1=\omega_2$.

14. The conditional equation for Ω_1, Ω_2 is

$$\nu(\omega_2^2-\Omega^2)+\mu(\omega_1^2-\Omega^2)+A_1A_2\omega_2^2g/a\omega_1=0 \quad . \quad . \quad (27)$$

This may be written

$$g=-\frac{a\omega_1}{A_1A_2\Omega^2}\;\frac{\mu(\omega_1^2-\Omega^2)^2+\frac{1}{2}\sigma^2\nu\omega_1^2\Omega^2}{\omega_1^2(1+\frac{1}{2}\sigma^2)-\Omega^2} \quad . \quad . \quad (28)$$

by the substitution

$$\omega_2^2=\Omega^2\{\omega_1^2(1+\tfrac{1}{2}\sigma^2)-\Omega^2\}/(\omega_1^2-\Omega^2). \quad . \quad . \quad . \quad (29)$$

Since (by (29)) the denominator of (28) has always the same sign as $\omega_1^2-\Omega^2$, and since the two roots are on either side of ω_1, it follows that A_1 and A_2 must have the same sign for the higher frequency and opposite signs for the lower frequency in order to make the condition (28) possible. Thus, with the winding of Fig. 1, it is always the oscillation of lower frequency which is set up. If the capacity of the condenser is gradually increased, thus diminishing ω_2, the oscillation frequency remains near to and slightly below $\omega_1/2\pi$ so long as ω_2 is greater than ω_1. When ω_2 is less than ω_1, the oscillation frequency remains near to and slightly below $\omega_2/2\pi$.

The effect may be shown graphically by plotting (25) with ω_2^2 as abscissæ and ω^2 as ordinates. The result is a hyperbola whose asymptotes are $\omega^2=\omega_1^2$, $\omega^2=\omega_2^2+\frac{1}{2}\sigma^2\omega_1^2$. That branch of the hyberbola which passes through the origin gives the frequencies of the oscillations which may be set up when the coils are connected as in Fig. 1.

The comparative independence of the oscillation frequency of the value of the capacity used may account for the known independence of frequency of a maintained fork of such factors as variation of the steady anode voltage.

In electrically-excited oscillations the change in the anode battery will alter (by its change in earth capacity) the effective capacity across the anode circuit. This will have its full effect on frequency in electrically coupled circuits, but practically none in the case of the vibrating fork.

15. The condition for easiest maintenance may be obtained from (27) as follows :—

When Ω is nearly equal to ω_2, the term involving ν may be neglected and

$$g \doteq \frac{a}{A_1A_2} \frac{R}{L}(1-\omega_1^2/\omega_2^2). \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (30)$$

For similar anode and grid coils, a/A_1A_2 is the equivalent capacity due to the reaction of the system on one coil, and is high (100 microfarads in the typical case). The oscillations are therefore difficult to maintain. When Ω is in the neighbourhood of ω_1, put $\omega_1^2-\Omega^2=v$ in (28), and retain only the first order term. Then g is of the form

$$g=K(v^2+m^2)/(v+n), \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (31)$$

in which

$$K=-a\mu/A_1A_2\omega_1, \quad m^2=\tfrac{1}{2}\sigma^2\omega_1^4\nu/\mu, \quad n=\tfrac{1}{2}\sigma^2\omega_1^2.$$

It is a minimum in regard to variations in v when v satisfies the quadratic

$$v^2+2vn-m^2=0 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (32)$$

and then

$$g=2Kv=g_1 \text{ (say)} \quad . \quad . \quad . \quad . \quad . \quad . \quad (33)$$

The electrical frequency required for this is found by substituting v for $\omega_1^2-\omega^2$ in (25). This gives

$$\omega^2=\omega_1^2(1+n/v). \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (34)$$

Equation (32) has two roots of opposite sign, of which the positive one has the smaller absolute value, and will therefore give the smallest possible value of g in (33). Since it is positive, K must also be positive, or the two coils must be oppositely wound. Further, from (31), since when no condenser is present, $v=0$,

$$Km^2/n=g_0=-\beta/A_1A_2 \text{ (by (5))},$$

as might otherwise be determined by substitution of the values of K, m, n. Using this in (33) to eliminate K,

$$g_1=2g_0vn/m^2=2g_0(v/n)(n/m)^2$$

Now (32) may be written

$$(v/n)^2+2(v/n)+m^2/n^2=0 \quad . \quad . \quad . \quad . \quad . \quad . \quad (35)$$

so that g_1/g_0 is a function of the single variable $(n/m)^2$. Denoting this by η and using the positive root of (35)

$$g_1=2g_0(\sqrt{\eta+\eta^2}-n), \qquad (36)$$

while (34) becomes

$$\omega_2^2=\omega_1^2(1+\eta+\sqrt{\eta+\eta^2}). \qquad (37)$$

The oscillation frequency is most simply expressed by

$$\Omega_1^2=\omega_1^2(1+\tfrac{1}{2}g_1/g_0 \,.\, \beta L/aR). \qquad (38)$$

In terms of the fundamental constants

$$\eta=R(R+A^2/\beta)/L^2\omega_1^2. \qquad (39)$$

It may thus be expressed in terms of two phase angles, θ and φ, φ being that of the coil alone and θ the modified phase angle obtained by adding the leakage resistance of the equivalent condenser of Fig. 2 to the resistance of the coil. In fact, θ is the actual phase angle of the coil when fed with currents of frequency $\omega_1/2\pi$, the reaction of the moving system being taken into account.

In terms of these phase angles

$$\eta=\cot\theta\cot\varphi \qquad (39\text{A})$$

16. In the typical case (Sections 3 and 9) $\eta=0{\cdot}056$, so that $g_1=0{\cdot}38\ g_0$, $\omega_2=1{\cdot}14\ \omega_1$; and since $\beta L/aR=0{\cdot}002$, $\Omega_1=(1-0{\cdot}00019)\omega_1$. Since $1/g_0=5{,}000$ ohms, $1/g_1=13{,}000$ ohms, so that 1 volt on the grid must now be capable of developing 77 microamperes. The three cases treated may be summarised thus:—

Condensers used.	g min microamperes per volt.	Optimum electrical frequency.	Oscillating frequency.
0	200	...	1000·0
1	77	1 140	999·8
2	20	1 000	1000·0

The value of the added condensers is most marked when η is small. When this is so, (36) and (37) may be written

$$g_1=2g_0\sqrt{\eta},\ \omega_2=\omega_1(1+\tfrac{1}{2}\sqrt{\eta}) \qquad (40)$$

For higher values of η, the values of g_1/g_0 and ω_1^2/ω_2^2 may be exhibited graphically. This is done in Fig. 4 up to $\eta=2$. When $\eta>2$, the addition of a condenser is of little value.

17. If in (25) and (27) we make the substitutions

$$x = \omega_1^2/\omega_2^2, \quad y = \omega_1^2/\omega^2$$

we obtain
$$y = x/y = 1 + px \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (41)$$
and
$$q(x-1) - x/y + 1 + z = 0 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (42)$$
in which
$$p = 1 + \tfrac{1}{2}\sigma^2, \quad q = \alpha R/(\alpha R + \beta L), \quad z = A_1 A_2 L g \alpha/(\alpha R + \beta L).$$
Eliminating x/y between (41) and (42), the latter may be replaced by
$$y + z = (p-q)x + q \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (42\text{A})$$

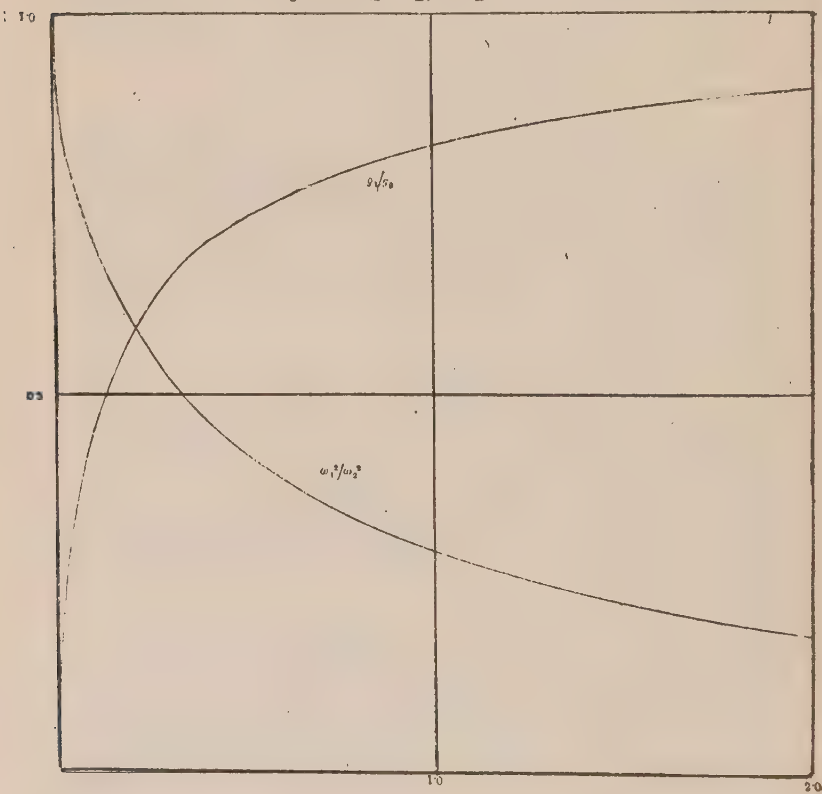


FIG. 4.

In these forms, the frequency equation (41) and the conditional equation (42A) admit of a simple graphical solution which holds for all values of the constants.

Taking axes x and y, draw the hyperbola (41) and on the same diagram draw the straight line $y = (p-q)x + q$. Then the ordinates of the hyperbola are proportional to the squares of the oscillation periods and the abscissæ to the capacity

required to produce them. The vertical distances of the straight line from the hyperbola gives the values of z, being positive above and negative below the line. Since z is proportional to g, its smallness is a measure of the ease of maintenance of the corresponding oscillation. The upper branch of the hyperbola gives the periods when the coils are oppositely wound and the lower branch when they are similarly wound.

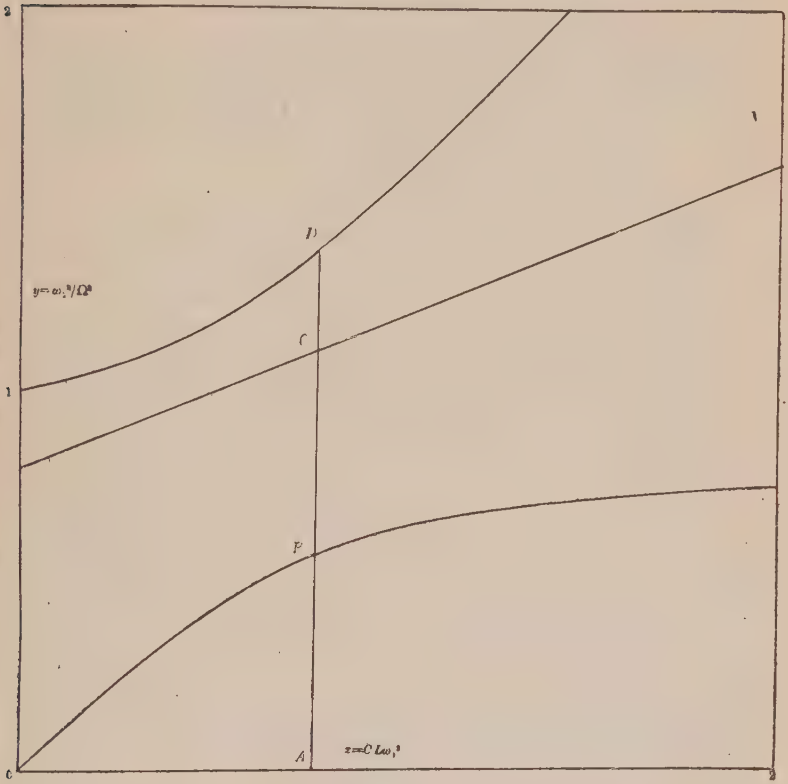


FIG. 5.

This construction is made in Fig. 5 for the hypothetical case $p=1{\cdot}2$, $q=0{\cdot}8$. If OA represents the capacity of the condenser, then the square of the two possible periods are represented by AB and AD, the former occurring when the coils are similarly wound. The values of g required are proportional to BC and CD respectively. The figure shows clearly the two optimum capacities and the advantage of using opposite windings.

18. In developing the theory, the effect of fluctuations of the anode voltage upon the anode current has been ignored. If v and v' be the variations in voltage of the grid and anode, the current leaving the valve by the anode varies by an amount

$$i_a = -gv - g'v'.$$

This relation may be written

$$i_a + g'v' = -gv,$$

which shows that if we still suppose the anode to deliver a current $-gv$, but shunt the anode circuits by a conductance g', the current flowing in the anode circuit will have the true value i_a. The effect of anode voltage upon anode current is exactly as if such a shunt were present.

ABSTRACT.

This Paper gives a mathematical analysis of the arrangement, previously described by Eccles, whereby the vibrations of a tuning fork are maintained by means of a triode.

In the simplest case, the prongs of the fork vibrate in the field of a permanent magnet, on the pole-pieces of which are wound coils connected in the grid and anode circuits of the valve respectively. From a consideration of the energy relations of the system, it is shown that for the maintenance of oscillations (a) the grid and anode coils must be wound in opposite directions, (b) there is a certain minimum value of the slope of the grid voltage-anode current characteristic of the valve at its working point.

Following this, the case in which condensers are connected across the two coils is treated on the principle of replacing the dynamical system by its electrical equivalent. In the particular example of similar anode and grid circuits, this system gives rise to three possible oscillations, one with the coils similarly wound and two with the coils oppositely wound. The conditions for maintenance are worked out, and the variation of the obtained frequency in the neighbourhood of the natural frequencies of both the dynamical and electrical systems is illustrated diagrammatically.

Finally, the case in which only one condenser is used is analysed, and a table is given showing the condition of maintenance and the oscillating frequency in the three typical examples.

DISCUSSION.

Dr. VINCENT said the question was one that would become of increasing importance. He would not be surprised to see instruments with valve-maintained vibrations taking the place of pianos.

Prof. FORTESCUE said the results were of importance apart from the application to tuning forks. In paragraph 5 the analogy between the electrical and mechanical system was enunciated ; but later on, at the end of paragraph 14, they are said to behave differently under alterations in the voltage of the anode battery. He did not see how this should be. The calculations referred to a circuit of relatively low resistance. In practice, the pole-piece coils must be of high resistance. The wide range of tuning is probably due to this. Equation 12 and the results deduced from it were similar to those obtained by other workers, and it would be useful to have them all collected and put in the same notation.

XXXII. *Radiation and Convection from Heated Surfaces.* By

MAJOR T. BARRATT, *D.Sc., A.R.C.S.,* and A. J. SCOTT, *B.A., B.Sc.*

RECEIVED MARCH 22, 1920.

I. OBJECTS OF EXPERIMENT.

THE question of the relative and absolute amounts of radiation and convection from heated surfaces of various shapes and sizes in a fluid—especially in air—is important from its intrinsic interest as well as from its commercial applications. The latter include hot wire ammeters, voltmeters, wattmeters, and anemometers,* psuchrainometers,† tube boilers and radiators.

In the following investigation "convection" is assumed to be that portion of the heat emitted from a surface, which is carried away by and in the direction of the cooling fluid; while "radiation" is emitted equally in all directions from a homogeneous heated surface.

Newton's law that the rate of cooling of a hot body placed in a current of air moving with uniform speed is proportional to its excess of temperature over that of the moving fluid has been shown experimentally to be approximately true in many cases, *e.g.*, for spheres heated in air to various temperatures, at various pressures,‡ and in currents at different speeds,§ and for metallic wires heated in air by an electric current.‖ Experiments carried out about four years ago by one of us ¶ showed that in the case of wires of about 1 mm. in diameter raised 10°C. or 12°C. above the temperature of the air in a cylindrical enclosure surrounding them, radiation formed a comparatively small proportion of the total heat (2·5 to 4·5 per cent. if the wire was "bright"), most of the loss of heat being due to convection. Newton's law was found to be applicable within the limits of temperature employed. In those experiments and in the determinations described in the present Paper the "convection" was what

* U. Bordini, "Nuovo Cimento," Pisa, 3, pp. 241-283, April 1912. J. T. Morris, "Electrician," Oct. 4, 1912. L. V. King, Roy. Soc. "Phil. Trans.," 373, Nov. 12, 1914. *See also* a Paper on "A Directional Hot-wire Anemometer," by Mr. J. S. G. Thomas, read before the Physical Society, March 12, 1920.

† J. R. Milne, Journ. Scott. Met. Soc., XVI., XXIX., 1912.

‡ P. Compan, "Comptes Rend.," 133, p. 1813, 1901; and 134, p. 522, 1902.

§ Crichton Mitchell, Trans. Roy. Soc., Edin., XLI., 4, p. 39, 1900.

‖ Kennelly and Sanborn, Amer. Phil. Soc. "Proc.," 55-77, 1914.

¶ Proc. Phys. Soc., XXVIII., Dec. 1, 1915.

is usually termed "natural" or "free" convection, *i.e.*, that caused by the movement of the air due to the excess of temperature of the surface over that of the air surrounding it. The present experiments, however, differ from those of the former Paper in that the temperature difference is greater, the surfaces are larger, and they are not enclosed in an outer jacket. The surfaces tested are cylinders (including a flat surface) and spheres of different radii of curvature.

A mathematical basis for Newton's law in the case of convection of heat from a cylindrical surface by a fluid moving at right angles to the heated surface (*i.e.*, "forced" convection) has been given by A. Russell,* who also showed that the loss of heat per square centimetre must be inversely proportional to the square root of the diameter of the cylinder. The results of the experiments described below are in accord with the theory in the case of "natural" convection, *when convection alone is considered*, *i.e.*, when the heat due to radiation is subtracted from the total heat emitted. In the case of spheres the convection per unit area appears to be inversely proportional to the *cube root* of the diameter. (*See* Section V., Tables VIII. and IX.).

II. Theory and Method of Experiment.

The temperature of the surface was raised to about 100°C. by passing steam through the hollow cylinder or sphere.† The total heat lost from the surface (convection + radiation) was obtained from the equivalent mass of steam condensed, (1) when the surface was "bright," (2) when it was covered with a "dead black" varnish.

The relative amounts of radiation from "bright" and "black" surfaces were then measured by a thermopile in conjunction with a low resistance galvanometer. This ratio was sensibly the same whether the surface was plane (as in a "Leslie" cube) or curved (as in a spherical surface). As regards convection it was assumed (as shown by Dulong and Petit in their classical research on emission of heat from various surfaces) that the convection from a particular surface is unchanged by coating it with dead black, the increase in emission being in that case all due to increased radiation.

* A. Russell, Proc. Phys. Soc., XXII., p. 432, 1909.

† The method is somewhat similar to that employed by J. A. Hughes, as described in a Paper "On the Cooling of Cylinders in a Stream of Air," "Phil. Mag.," XXXI., pp. 118-130, Feb., 1916.

The exact temperature of each surface when steam was passing through the tubes was directly measured by means of a thermojunction of very thin copper and constantan wires (*see* Table I.). The junction was soldered into a "notch" made in the outside of the tube, the solder being then filed down to the same curvature as the rest of the surface. In the case of the glass spherical surface, the thermojunction was secured firmly in the notch by means of a thin strip of tin foil.

Now, assuming that the difference of the temperatures between the outside and the inside of the tube is given by $t_1-t_2=\frac{H \cdot d}{K \cdot A}$, where H is the heat emitted per second from area A of a tube of thickness d, we find that t_1-t_2 is of the order 0·006°C. It is by no means safe, however, to assume that the temperature of the inner surface of the copper is the same as that of the steam. Assumptions of the nature have, in fact, led to astonishing errors in some of the earlier determinations of coefficients of thermal conductivity of metallic conductors.

Let h_1, r_1, and c be the total heat, radiation, and convection respectively from the black surface per second, per square centimetre, per 1°C. excess temperature, and h_2, r_2, and c corresponding quantities for the bright surface, and let $h_1=ah_2$, i.e., $c+r_1=a(c+r_2)$. Further, let $r_1=br_2$. Then

$$\frac{r_2}{h_2}=\frac{a-1}{b-1} \qquad (1)$$

and

$$\frac{r_1}{h_1}=\frac{b(a-1)}{a(b-1)}. \qquad (2)$$

Equations (1) and (2) give the proportion of radiation to total heat lost for "bright" and black surfaces respectively; "a" being obtained from the total heat experiments and "b" from the "radiation" experiments. The actual amounts of "total heat," "radiation," and "convection" from each surface can thus be easily found.

III. Apparatus and Experimental Work.

The apparatus employed for measuring the "total heat" is sketched in Figs. 1 and 2.

The copper tube AB (Fig. 1) was suspended by two thin cords at a slope of about 1 in 7 from the horizontal, about

50 cm. from the ceiling of a small room free from draughts. (A few experiments were made with the tube vertical, the results being very slightly lower.) The dimensions of the room were $3 \cdot 5 \times 2 \cdot 5 \times 3 \cdot 5$ metres.

FIG. 1.—APPARATUS FOR MEASURING "TOTAL HEAT" FROM A CYLINDER.
AB = Copper tube ; V = Collecting vessel for condensed steam.
T = Water trap ; C = Tube from boiler ; t = Tube leading to condenser.

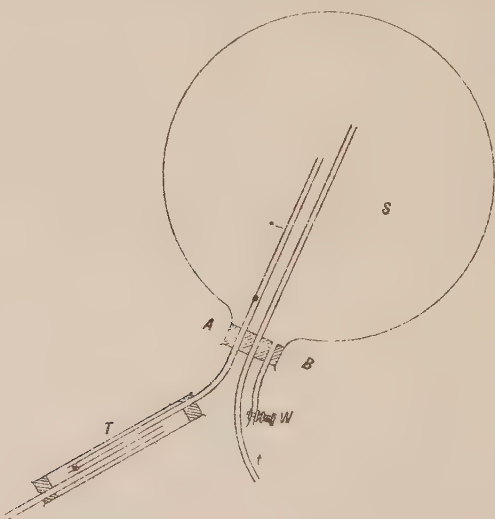


FIG. 2.—APPARATUS FOR MEASURING "TOTAL HEAT" FROM A SPHERE.
S = Sphere ; T = Water trap ;
t = Tube leading to condenser ;
W = Tube for collecting condensed steam.

Steam was passed through the tube from a small boiler some distance away and separated from the tube itself by a wooden screen. A water trap T prevented water particles being carried over into the tube. The leading tubes from

A to C were thickly lagged with cotton wool. The water condensed from the steam was collected in a thin metallic vessel V, and after conditions had become steady was run off and weighed at intervals usually of 10 minutes. Any uncondensed steam was conveyed by a tube t to a condenser about a metre away. The temperature of the ascending air was read by a thermometer placed some 75 cm. immediately below the tube. Experiments were conducted with the tube (*a*) bright, (*b*) black, (*c*) bright, and the average of (*a*) and (*c*) compared with (*b*).

At the end of a series of experiments (20 or more) the tube was sawn off at G about 1 cm. from the collecting vessel V and the latter directly connected with the steam supply by inserting the cork A into the end G. A series of similar readings was then taken in order to obtain the correction for loss of heat from the collecting vessel. The remaining portion of the tube $A\ G$ was that taken in the calculations as the emitting surface.

For the flat surface a flat copper vessel of dimensions approximately $70 \times 15 \times 1{\cdot}4$ cm. and total surface area 2443·6 sq. cm. was connected to the boiler, with the 15-cm. surface either horizontal or vertical, or at an angle inclined to the vertical. The results obtained showed too great a variation to form any definite conclusion. Alteration of slope often produced a great change in the total loss of heat. The irregularity of shape may have produced a considerable and irregular change in the flow of air currents. In all cases, however, the total loss of heat per square centimetre was greater than in the cylinder of largest radius of curvature considered.

The spherical surfaces employed (*see* Fig. 2) were thin glass flasks, which were coated externally with (*a*) aluminium varnish, (*b*) "dead black." Steam entered through a water trap T, the uncondensed portion leaving through tube t to a condenser, and the condensed steam being collected at intervals of 10 minutes from the tube W. Correction for loss of heat in this case is unnecessary, no "collecting vessel" being employed, and the water trap being close up to the spherical enclosure.

All parts below AB were lagged with cotton wool. Separate experiments in radiation were necessary to obtain the ratio for "aluminium" and "dead-black" surfaces. This was done by coating one half of the flask itself with aluminium

and dead-blacking the other half, and employing thermopile and galvanometer in the usual way.

IV. Experimental Results.

A. *Cylindrical Surfaces.*

(1) *Total losses of heat.*—Steam was passed for about an hour before any measurements were taken. The slope of the tube was about 1 in 7. The current of air upwards was at the rate of about 1 ft. per second as tested by tobacco smoke.

Table I. gives results for the four tubes and for the flat surface.

Table I.

	Tube 1.	Tube 2.	Tube 3.	Tube 4.	Flat Surface.
Length (cm.)	61·0	65·3	77·1	72·0	...
Diameter (cm.)	0·62	1·27	2·54	5·12	...
Surface area (sq. cm.)	118·9	262·2	616·4	1158·0	2444·0
Surface temperature (°C.)	97·7	98·3	99·3	99·65	99·7
Heat per sec. per sq. cm. per 1°C. excess	...	...	...	...	...
Calories $\times 10^{-6}$ black	495	362	280	222	317
Calories $\times 10^{-6}$ bright	432	285	208	155	215

In the last two lines of figures corrections have been made for the loss of heat from the collecting vessels (10 to 20 per cent.).

The degree of consistency of results is shown in Table II., obtained for the tube of diameter 1·27 cm.

Table II.

Tube bright.

Heat per sec. per sq. cm. per 1°C. excess. Calories $\times 10^{-7}$.	Temperature excess. °C.
2869	75·6
3012	78·0
2928	79·7
2882	79·8
2819	76·7
2780	80·2
2743	78·0
2751	79·3
Mean Result .... 2848×10^{-7} calories.	

(2) *Relative losses by radiation for "bright" and "black" surfaces.*—One side of a "Leslie" cube was bright copper, another was coated with the same dead black as used in the

total heat experiments. Boiling water was poured into the cube and the radiation measured alternately from "black" and "bright" surfaces. The mean ratio was found to be 5·37 : 1, *i.e.*, "b"=5·37.

(3) *Collected results for the four tubes.*

TABLE III.

Diameter cms.	Total loss of heat "h" per sq. cm. per sec. per 1°C. (calories × 10^{-6}).		Loss due to convection (c) (calories × 10^{-6}).	Radiation loss (calories × 10^{-6}).	
	Black (h_1)	Bright (h_2)		Black (r_1)	Bright (r_2)
0·62*	495	432	416·0	79·0	16·0
1·275	362	285	267·5	94·5	17·5
2·545	280	208	182·0	98·0	16·0
5·12	222	154	139·5	82·5	15·5

* A different "black" for which "b" = 5·0 was used with this tube.

Equations (1) and (2) give the following percentages for the radiation losses.

TABLE IV.

Diameter of tube. cms.	Percentage of radiation. Black surface.	Percentage of radiation. Bright surface.
0·62	16·0	3·7
1·275	26·1	6·1
2·545	35·0	7·7
5·12	37·1	10·1
Flat surface	39·5	10·9

B. *Flat Surfaces.*

Table V. gives collected results for the flat surface with its length nearly horizontal and breadth vertical.

TABLE V.

Total area of flat surface	2443·6 sq. cms.
Total loss of heat (h) { black	317 × 10^{-6} calories.
Total loss of heat (h) { bright	215 × 10^{-6} ,,
Loss due to convection	192 × 10^{-6} ,,
Loss due to radiation { black	127 × 10^{-6} ,,
Loss due to radiation { bright	23 × 10^{-6} ,,
Percentage loss due to radiation { black	39·5 per cent.
Percentage loss due to radiation { bright	10·9 per cent.

C. *Spherical Surfaces.*

(1) *Total losses of heat (radiation and convection).*

TABLE VI.

Diameter cms.	Effective area of surface. sq. cms.	Total loss of heat per sq. cm. (h) per 1°C. excess (calories × 10^{-6}).		Value of "a" $\left(=\frac{h_1}{h_2}\right)$
		Black.	Bright.	
7·55	182·0	405	311	1·30
10·07	314·3	371	288	1·29
14·0	611·7	343	257	1·33
16·5	849·2	333	247	1·34

(2) *Ratio of radiation losses (black : aluminium).*—A large spherical flask of diameter 14 cm. was coated on one side with aluminium paint, the other dead blacked. The value of "b" was found to be 2·37.

Taking the value of "a" from Table VI. we obtain the following values for convection and radiation for the spherical surfaces. (*See* Table VII.)

TABLE VII.

Radius cms.	Loss due to convection per sq. cm. per 1°C. excess. (calories × 10^{-6}).	Loss due to radiation per sq. cm. per 1°C. excess. (calories × 10^{-6}).		Percentage loss due to			
				Convection.		Radiation.	
		Al (r_1).	Black (r_2)	Al.	Black.	Al.	Black.
7·55	243	68	162	78·1	60·1	21·9	39·9
10·07	226	62	145	78·8	61·1	21·2	38·9
14·00	196	61	147	75·5	57·1	24·1	42·9
16·50	186	61	147	75·2	56·1	24·8	43·9

The percentage of radiation is seen to increase only slightly with increase of radius of curvature within the limits observed.

V. DISCUSSION OF RESULTS.

(*a*) *Comparison of "heat lost by convection" with the diameter of cylinder.*—It has been shown in Russell's Paper* that the convection of heat from a cylinder per square centimetre of surface must be inversely proportional to the square root of the diameter. The proof applies strictly only to the case of

* *Loc. cit.*

a cylinder immersed in a cooling fluid which moves with constant appreciable but not excessive velocity in a direction at right angles to the length of the cylinder. The fluid is supposed to be opaque to heat rays, non-compressible, of very low thermal conductivity and to possess no viscosity. The cases tested in the present Paper can only be expected, therefore, to give approximate agreement with this theory.

Table VIII. shows the relation actually found between the heat lost (by convection only) and the diameters of the cylinders. The product of the "convection" and the square

TABLE VIII.

Diameter d. cms.	$\sqrt{d}$.	Heat lost by convection. (c) (calories $\times 10^{-6}$).	$c\sqrt{d}$.
0·62	0·787	416·0	327
1·275	1·129	267·5	302
2·545	1·595	182·0	290
5·12	2·263	139·5	316

root of the diameter appears to be approximately constant, *i.e.*, $c \propto \frac{1}{\sqrt{d}}$ within limits of experimental error.

(*b*) *Convection from spherical surfaces.*—In the case of a *sphere* the figures indicate that the convection per square centimetre is inversely proportional to the *cube root* of the diameter. (*See* Table IX.)

TABLE IX.

Diameter d. cms.	Convection loss per sq. cm. (c). (calories $\times 10^{-6}$).	$C \times \sqrt[3]{d}$.
7·55	243	456
10·07	226	488
14·0	196	472
16·5	186	472

That is $c \propto \frac{1}{\sqrt[3]{d}}$.

(*c*) *Losses by radiation.*—The percentage radiation losses as given by the experiments described above are considerably higher than those obtained in a previous paper for wires of 1 mm. diameter. This is partly accounted for by the greater difference of temperature between the hot surface and the ascending gas (80°C. compared with 10°C. or 12°C.), but

chiefly because the total losses are smaller per square centimetre, while the radiation remains approximately constant Radiation losses as a percentage of "total heat" emitted are given in Table X.

TABLE X.

Surface and radius of curvature.	Percentage of radiation loss.	
	Bright.	Black.
Cylinders 0·32 cm.	3·7	16·0
0·64	6·1	26·1
1·28	7·7	35·0
2·56	10·1	37·1
Flat surface	10·9	39·5
Spheres 3·77 cm.	21·9	39·9
5·03	21·2	38·9
7·00	24·1	42·9
8·25	24·8	43·9

The radiation is given—approximately at any rate—by Stefan's fourth power law, $R=k(\theta^4-\theta_0^4)$. It is interesting to compare the radiation losses as given by this equation for the two cases,

(*a*) where $\theta=100°C.$; $\theta_0=10°C.$,

and (*b*) where $\theta=20°C.$; $\theta_0=10°C.$

The ratio is $\dfrac{373^4-283^4}{293^4-283^4}=13{\cdot}5.$

Thus the increase in radiation is only $13\frac{1}{2}$ times more in the first case than in the second.

At the same time the convection (by Newton's law) would increase 9 times.

In cases where the temperature excess is not very high and the percentage of radiation is small (as in wires and narrow tubes), it follows that little error is made in assuming the linear law for *the total loss of heat* from the surface. Results given in Tables III. and VII. indicate that the loss of heat per square centimetre by radiation alone is independent of the curvature of the surface. That is to say that the increase of loss of heat due to decrease of radius of curvature is entirely due to increase in convection.

In the case of the flat surface the radiation per square centimetre appears to be greater than in the curved surfaces, but owing to the variation in results obtained for slight changes of slope, &c., no great weight can be attached to this.

The experiments were performed at the Wandsworth Technical Institute, most of them about four years ago, but owing to military duties publication has been delayed until now.

Summary.

An experimental determination has been made of the amounts of "natural" convection and radiation from cylindrical and spherical surfaces heated in air at atmospheric temperature and pressure to a temperature of about 100°C.

It is shown that the convection alone is inversely proportional to the square root of the diameter of a cylinder, and to the cube root of the diameter of a sphere.

The percentage of radiation to total heat emitted is given in the following table:—

Surface.	Diameter cms.	Percentage of radiation.	
		Bright.	Black.
Cylinders	0·32	3·7	16·0
	1·275	6·1	26·1
	2·545	7·7	35·0
	5·12	10·1	37·1
Flat surface	...	10·9	39·5
Spheres	7·55	21·9	39·9
	10·07	21·2	38·9
	14·00	24·1	42·9
	16·50	24·8	43·9

ABSTRACT.

The relative and absolute amounts of radiation and convection from surfaces heated to about 100°C. in air were measured as follows: steam was passed through the cylinders or spheres, the surface temperature being measured by a thermo-junction. The total amount of heat lost from the surface was determined from the equivalent mass of steam condensed. This was done with the surface (a) "bright," (b) "dead-black." The relative amounts of radiation alone from these two surfaces were then found with the aid of a thermopile. If h_1 is the total heat lost, and r_1 the radiation, per square centimetre per second per 1°C. excess of temperature for the "black" surface; h_2 and r_2 corresponding quantities for the "bright" surface; and c the convection in each case, then if $h_1=ah_2$, $r=br_2$,

$$\frac{r_2}{h_2}=\frac{a-1}{b-1}, \quad \frac{r_1}{h_1}=\frac{b(a-1)}{a(b-1)}.$$

The numerical values of the radiation, convection and total heat are then easily calculable, and are given in tables. It is found that the amount of *convection* per square centimetre is inversely proportional to the square root of the diameter of a cylinder, and to the cube root of the diameter of a sphere.

DISCUSSION.

Mr. C. R. DARLING pointed out that the numerical results contained some considerable discrepancies. For instance, in Table 2, for an excess of 75·6 deg. the heat loss was 2,869 units, while with a higher temperature excess of 80·2 deg. it was only 2,780. Something must be wrong, and he distrusted the use of steam condensation methods of measuring the heat lost. Water traps were never satisfactory. Why not use electrical heating ? A second point was the method of measuring the surface temperature by inserting a thermo-junction in a little notch. Some experiments he had once made showed that the conditions just under the surface differed considerably from those at the actual surface when heat is escaping. Had the author tested the results mentioned by Langmuir ("Journ." Am. Electro-Chem. Soc., Vol. 23, 1913) ? Langmuir states that the convection loss is equal to the heat conducted through a layer of air of particular thickness surrounding the hot body. Further, the loss from a plane disc is 10 per cent. more when the disc is horizontal than when it is vertical. In the horizontal case the loss from the lower surface is 50 per cent. less than from the upper. Langmuir found that the phenomena of natural convection did not agree with Russell's formulæ, but forced convection did.

Dr. EZER GRIFFITHS thought the Paper of interest insomuch that it contributed some data for cylinders of moderate diameters and for spheres. The experimental work of Langmuir referred to by the last speaker did not cover the same ground, since it dealt only with fine wires and one flat surface. He agreed that the method of steam heating was not altogether satisfactory, although most of the data relating to steam pipes had been obtained by means of it. When electrical methods were employed the difficulty was the large amount of energy which had to be supplied. For example, in some experiments on convection loss now in progress it was necessary to supply about 18 kw. of electrical energy to heat up a cylinder 9 in. in diameter by about 4 ft. long to a temperature of about 450°C. He did not think that the experiments of Dr. Barratt proved conclusively the relation between diameter and convection loss per unit area given in the Paper. It was a well-known fact that the change of heat loss with diameter was greatest with cylinders of small diameter, such as wires, but above about 3 in. in diameter the change was small and the form of the curve connecting heat loss (convection per unit area), and diameter for the complete range of diameters (wires of 2 mils to cylinders 9 in. diameter) was of the form which he illustrated on the board.

Capt. DUNSHEATH said that at present millions of pounds were invested in cables for conveying electric power, and more information on the heat loss from such cables would be of the greatest practical value in determining the necessary thickness of copper to employ. The present experiments, however, did not carry us far in this respect, as the cable is rarely suspended in free air. It would be useful if the experiments could be carried out with the cylinder in contact with a plane or lying in a duct. As regards the measurement of surface temperature, he thought the conduction along the wires of the couple would affect this unless they were arranged to be in thermal contact with the surface for some distance from the junction. How was the effects of draughts eliminated ? Were any steps taken to measure the air currents existing before the cylinders were heated ?

Mr. G. D. WEST mentioned the importance of the inclination of the surfaces to the vertical. With wires in tubes a great deal depended on whether the tubes are horizontal or vertical. If the author could give us data on this point, we should be a step nearer measuring the conductivity of the surrounding gas, which, at low pressures, is more important than convection.

Mr. F. E. SMITH agreed with what had been said regarding the inefficiency of steam traps. It appeared to him that measurements of this type could be carried out by using very thin walled tubes conveying an electric current.

By measuring the resistance of the tube, both the temperature and the energy consumed could be determined.

Dr. RUSSELL (communicated): As a knowledge of the relative amounts of heat convected and radiated from hot bodies is of great practical value to the engineer, the authors are to be congratulated on having made a notable contribution to our knowledge of the subject. It is satisfactory to find that the heat convected from the cylinders used by the authors varies approximately inversely as the square root of their diameters, although only free convection is considered. I think that we may infer that the mean velocities of the convection currents of air round all the cylinders is approximately the same. I was surprised at the large amount of heat convected per unit area from the spherical glass vessels as compared with that convected from cylinders of equal diameter. For the same difference of temperature between the bodies and the air I should have expected, from theoretical considerations, that the heat convected per square centimetre from a cylinder would be $2\sqrt{2}/\pi$—that is, 0·9 of that convected from a sphere of equal diameter. It appears from the authors' results that the heat convected from a cylinder of 7·55 cm. diameter is only about 0·45 of that convected from a sphere of the same diameter—that is, it is only about half the theoretical value. On certain theoretical assumptions, also, the heat carried away per square centimetre from a sphere by forced convection varies inversely as the square root, and not inversely as the cube root of the diameter. Further research on the cooling of spheres, more especially with forced draught, would be very interesting. I should expect that the heat convected per unit area would vary as the square root of the velocity of the draught, and inversely as the square root of the diameter. The heat also convected per unit area from a sphere should be about 10 per cent. greater than that convected per unit area from a cylinder of the same diameter placed with its axis perpendicular to the same draught.

Dr. BARRATT communicated the following reply to the points raised by speakers: The "heat loss" mentioned by Mr. C. R. Darling is in each case the result *per degree excess.* My reason for employing a steam condensation method was for the purpose of simplicity in apparatus. As regards "surface temperature" (criticised also by Capt. Dunsheath), each temperature measured was in the region of 100°C., and was directly compared with the temperature of steam at atmospheric pressure. I am convinced the temperatures measured were pretty accurate. I read Langmuir's results some years ago. Natural convection depends so much on the size and disposition of apparatus, and the nature of the surroundings, that the problem of a flat surface appears to be almost insoluble experimentally. In reply to Dr. Ezer Griffiths' remarks as to the change of heat loss with curvature in cylinders of large diameter, it is worthy of notice that in this case the radiation loss becomes relatively very large. When the convection loss alone is considered (this being thoretically nil for an infinite flat surface), the inverse square root law appears to be approximately applicable. No special precautions were taken to exclude draughts beyond those mentioned in the Paper, and these probably accounted to a great extent for the discrepancies shown in the results. Mr. F. E. Smith's suggestion is a good one if one could employ tubes of a pure metal (*e.g.*, platinum), where the temperature resistance curve remains constant. The question raised by Mr. G. D. West of convection losses from wires in tubes is a difficult one to test experimentally, owing to the impossibility of obtaining constancy in the natural convection stream.

THE FIFTH GUTHRIE LECTURE.

The Anomaly of the Nickel-Steels.

Delivered by DR. CHARLES EDOUARD GUILLAUME, *Director of the Bureau International des Poids et Mesures, Sèvres.**

APRIL 23, 1920.

Discovery of the Anomaly.

IT is close on 30 years since John Hopkinson—whose tragic death still remains to us a sad memory—discovered a remarkable fact: a specimen of an alloy of iron and nickel containing 24 per cent. of nickel was soft and non-magnetic; but, on plunging it into solid carbon dioxide, it became hard and magnetic, its volume undergoing at the same time an increase of about 2 per cent.

This steel might be said to be of a decidedly anti-metrological character; and, so far as the purpose of my researches at the Bureau International des Poids et Mesures was concerned, I naturally sought to give a wide berth to all phenomena of the kind.

My investigations had for their object the perfecting of standards of length. The General Conference of 1889 had distributed a number of admirable standards amongst those States represented in the Metric Convention. These standards were constructed of the platinum-iridium alloy discovered by Henri Sainte Claire Deville, the preparation of which was entrusted to Johnson Matthey. But the great cost of these standards debarred their use for most requirements; whilst, on the other hand, the usual standards were far from reliable.

I therefore undertook the search for a metal or alloy which, whilst little inferior in general metrological qualities to platinum-iridium, should be considerably lower in price.

A preliminary investigation resulted in establishing the remarkable metrological properties of pure nickel; and, even to this day, it is this metal to which we have recourse when in need of a standard which shall be unoxidisable, invariable with time, rigid, and of moderate expansibility. A difficulty, however, deterred me from using it in all cases. The chief desideratum was to produce a geodesic standard of a length of four metres; but no firm of nickel manufacturers would undertake to supply a bar of that length perfectly sound and free from cracks.

* The Lecture was delivered in English. The translation of the French M., made by Dr. D. Owen, has had the advantage of the Author's perusal.

In 1895, M. J. R. Benoit, at the request of the Technical Section of Artillery at Paris, was engaged in the examination of a standard prepared from an iron alloy containing 22 per cent. of nickel and 3 per cent. of chromium. He found it to possess a coefficient of expansion approximately equal to that of brass. This alloy was non-magnetic, and appeared almost certainly to belong to the same class as Hopkinson's steel before its transformation by cooling. But the anomaly discovered by M. Benoit, although of great interest from the physical point of view, offered no particular prospect of advantage to metrology.

The question took a new aspect when, in the following year, I came across a new and quite unexpected fact, closely connected with those to which I have alluded. A bar of steel containing 30 per cent. of nickel, received at the Bureau International, was found to have a coefficient of expansion about two-thirds of that of platinum, and at the same time to be magnetic. Thus opened a promising line of investigation, which was eagerly pursued.

The question of expansibility is fundamental in metrology. Before the discovery of the anomaly just stated, every physicist would have affirmed that it was hopeless to attempt to solve the problem of finding alloys of a much lower coefficient of expansion than the values then known, since the rule of mixtures was regarded as holding practically exactly in all cases.

My first care was to verify the trend of the variation of expansibility of these alloys as a function of chemical composition. This precaution was not without warrant, in view of the possibility of a discontinuity occurring in the region between the non-magnetic alloy containing 22 per cent. of nickel, and the magnetic alloy containing 30 per cent. Experiments made on two alloys, on opposite sides of the last mentioned as regards nickel content, and which were supplied by the Company of Commentry-Fourchambault et Decazeville, served to establish the fact of continuity.

Classification by means of Magnetic Properties.

The precise measurement of the coefficient of expansion is a long and delicate one. Since one was in presence of an anomaly affecting, doubtless, all the properties of the new alloys, it became necessary to seek for methods of greater convenience. The examination of the magnetic susceptibility naturally suggested itself; where it is a question merely of

ascertaining the presence or absence of ferro-magnetism, this test is of the simplest. The outcome of the study of this effect was the recognition of the existence of two distinct transformations: the one irreversible, being that which Hopkinson discovered; the other reversible, and constituting a new phenomenon.

Other observers, notably MM. Osmond, Louis Dumas, Pierre Weiss and his pupils, Nagaoka, and Honda have worked out the details.

A simple diagram enables us to see at a glance the complete course of transformations of the series of ferro-nickels as related to their magnetic properties.

Commencing with pure iron, the variations resolve themselves along two branches which gradually get further apart

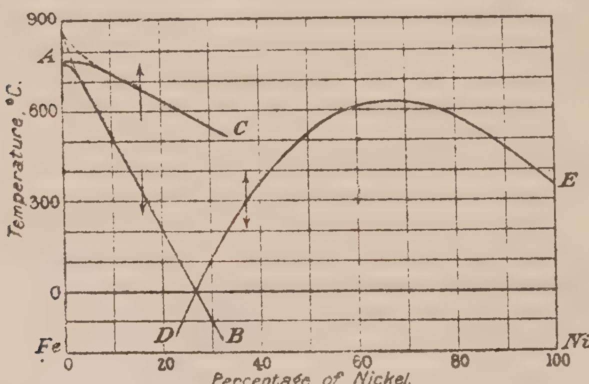


FIG. 1.

Temperature of magnetic transformation of the nickel-steels as a function of composition. The two branches AC and AB refer to the irreversible state; the single branch DE to the reversible state.

(Fig. 1). At the intersection, from top to bottom, of the lower curve, magnetism appears, and increases up to a certain limit. On reheating the alloy, it begins to decrease at a given temperature, and definitely disappears along the upper curve. On the contrary, the passage along the single branch of the curve denotes, in the case of alloys of higher nickel content, that the appearance of ferro-magnetic properties on cooling, and their disappearance with rise of temperature, occur at one and the same temperature.

The point of crossing of the curves of the two categories has a special significance; additions of carbon, chromium, man-

ganese* lower markedly the temperature of an irreversible transformation, but have much less influence on reversible transformations. It is thus possible to follow a reversible transformation in a normally irreversible region. At a time when the only copious source of liquid air available was at the Royal Institution, I succeeded, thanks to the courtesy of Sir James Dewar, in demonstrating the existence of reversible magnetic properties in the alloys containing 22 per cent. Ni and 3 per cent. Cr.

On the other hand, on the right of the crossing a moderate cooling still leaves the magnetic condition completely reversible; whilst a more considerable cooling brings about the transformation and renders it irreversible.

Let us now suppose a third axis perpendicular to the other two, along which is plotted the value of the susceptibility;

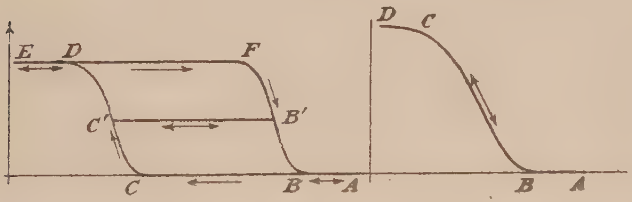


FIG. 2.

Variation of magnetic susceptibility of nickel-steels with temperature, in the irreversible and reversible regions respectively. (Ordinates denote susceptibility, abscissæ temperature.)

and, in the solid diagram thus obtained let a vertical section be taken. The variation of susceptibility with temperature will have, in the irreversible region, the slope given by the curves $ABCDEF$, and in the reversible region that represented by the unique curve $ABCD$ (Fig. 2).

One can, in the case of the irreversible condition, arrest the cooling at any moment, then reheat the alloys, thus conserving the magnetic properties constant, as shown by the line $C'B'$ (Fig. 2).

All the properties of the alloys with which we are concerned are bound up with these transformations.

* Manganese, alone, lowers the temperature of transformation of the alloys which it forms with iron; on this property depends the practical realisation, by Sir Robert Hadfield, of the manganese steels, with their very remarkable properties.

The clue being thus found, I gave my utmost efforts to the study of change of volume, which constitutes the point of capital metrological interest of these alloys.

Changes of Volume.

Irreversible Changes.—Knowledge of the irreversible changes which these alloys undergo was necessary, particularly with a view to fixing the limiting value. I have studied their different aspects in the binary alloys of nickel-iron, and their ternary alloys with chromium, copper, &c. It has only been possible for me to trace the lower part of the cycle ; complete cycles were realised much later by M. P. Chevenard.

When a rod of an irreversible alloy is cooled from a high temperature, it contracts according to a straight line law, as

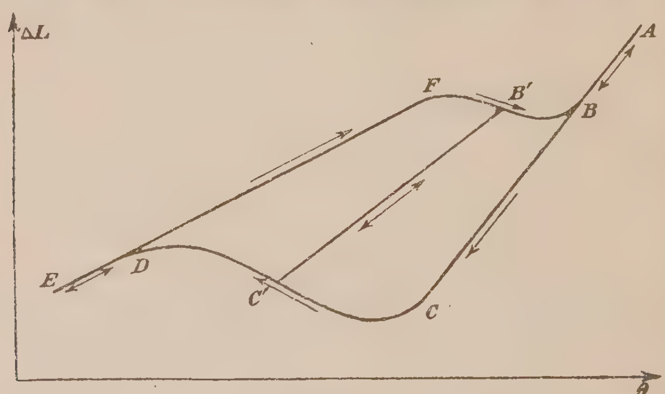


Fig. 3.

Expansion curve of a nickel-steel in the irreversible state.

seen in ABC (Fig. 3) ; at a certain temperature the contraction becomes less rapid, and finally a progressive increase of volume sets in, which goes on steadily until the transformation is complete. After that, the contraction resumes its regular reversible path DE.

If the rod is again heated, it is observed to expand along EDF, and then contraction sets in, which restores it to the line AC. If the cooling is arrested at the point C', and the bar reheated, the changes that occur at first follow the line $C'B'$, and then continue along the curve $B'BA$, just as in the preceding case. The inclination of the line AC is about 18×10^{-6}, and that of EF between 10 and 11×10^{-6}. The former figure

represents the expansibility of a non-magnetic alloy of iron-nickel-chromium; the latter the value in the case of the ordinary steels. Between these values one can obtain any intermediate rate of expansion by simply arresting the transformation at a suitable point on one or other of the branches *CD* or *FB*.

Along the curve *AC* the steel is in a state which is stable from the high temperature downwards (stable à chaud); along *EDF* it is in a state stable from low temperature upwards (stable à froid).

Reversible Changes.—Experiments on the reversible alloys have been much more extensive.

A preliminary investigation was made to establish the curve of the anomaly as a function of the nickel content, without any regard to the presence in slightly variable quantity of manganese, carbon and silicon—which left the curves to a certain extent indefinite. This was followed by a study of the series of alloys containing proportions of manganese and of carbon up to the maximum limit attainable. The coefficients as affected by the addition of these elements having thus been determined over the whole range of nickel content, it became possible to reduce the results to the special case of alloys in which the additions are in the uniform percentage proportions 0·4 Mn, 0·1 C, and which I have called "typical alloys."

These alloys, rolled when hot and allowed to cool in air, may be said to be "in the natural state."

The expansion of a rod being represented by the equation $l_\theta = l_0\,(1+\alpha\theta+\beta\theta^2)$, we shall call the quantity $\alpha_\theta = \alpha + 2\beta\theta$ the *true* coefficient of expansion at $\theta°$; it is also the *mean* coefficient of expansion between 0° and $2\theta°$. We shall term β the quadratic or curvature coefficient.

The expansion of most of the metals is well represented, over a large range of temperature, by an equation of the form given above; in the case of the alloys with which we are most concerned, the same law applies sufficiently well over the narrow range, 0°-38°, within which I have worked with the comparator; but when the range is extended it soon becomes evident that an equation of the second degree insufficiently represents the facts. It is, however, convenient to preserve the same form of equation, attributing to β a value varying with temperature, and equal to half the curvature at any point of the curve of expansion.

The values of α_{20} and β_{20} for the typical reversible nickel-steels in the natural state are represented by the curves of Figs. 4 and 5. The straight lines AB connect the values of

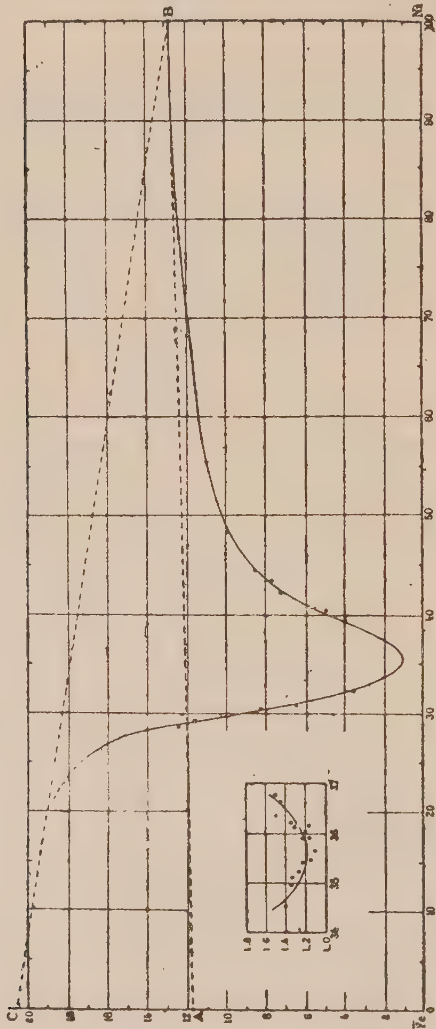


FIG. 4.

True expansibilities at 20 °C. (in millionths) of the typical alloys of iron and nickel (containing 0·4 Mn and 0·1 C per cent.). Abscissæ denote the percentage of Ni. The inset represents the region of the minimum to a larger scale.

these coefficients for pure iron and nickel in the condition which is stable at low temperature, and thus make evident the magnitude of the anomaly of expansion. In regard to β, this anomaly is positive at first, then becomes negative. The

same appears at first sight to be the case with α; but on tracing the line CB, which starts at the value which this coefficient possesses in the case of gamma iron, the anomaly is

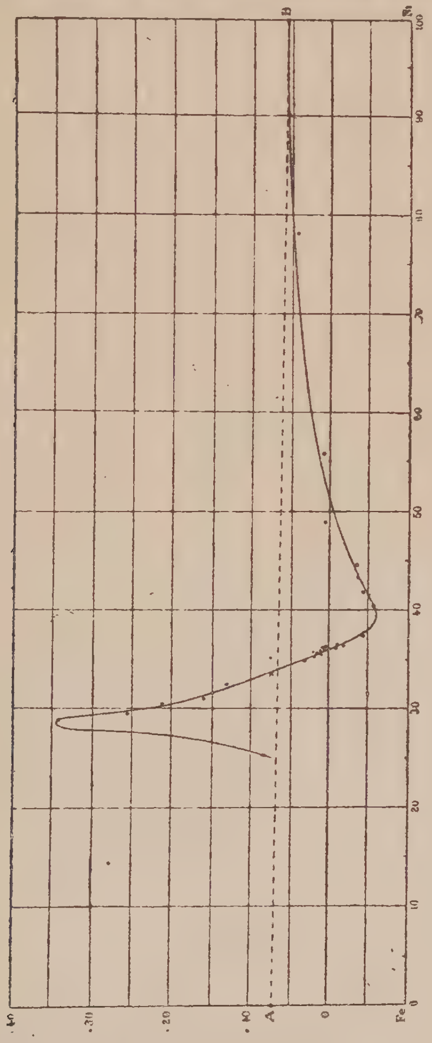


Fig. 5.

Values of the coefficient $10^9\,\beta$ of the typical alloys of iron and nickel (containing 0·4 Mn and 0·1C per cent.).

seen to be entirely negative. It will be shown later that the anomaly receives its full explanation in terms of this last fact. It may be observed, further, that the expansibilities of the irreversible alloys are limited by the straight lines AB and CD.

The anomaly is of considerable range, since the expansibility of the alloys varies in the extreme proportion of 1 to 15, the value at the lower limit being only a quarter of the least coefficient possessed by a metal, namely, that of tungsten. It may also be remarked, as a practical consideration, that these feeble expansibilities are obtained with the cheap metals : and that these provide a perfectly continuous scale, in contrast to that of the discontinuous and costly group of metals, such as iridium, tantalum and tungsten.

The lowest value ($\alpha = 1{\cdot}19 \times 10^{-6}$) possessed by the typical alloys corresponds to a content of 35·6 per cent. of Ni.

The generic name *Invar* (diminutive of invariable) has been given to alloys possessing expansibilities in the neighbourhood of the minimum value.

There is no object in assigning a precise value to the coordinates of the minimum point. As a matter of fact, the values quoted refer to an alloy in which the proportion of other elements was fixed arbitrarily, of the order of the average quantities present in industrial alloys ; we shall see later that the proportions of additional metals present are of considerable influence on the position of the minimum. On the other hand, all treatment, thermal or mechanical, which the alloys undergo, modifies their expansibility ; the value rising in case of heating followed by slow cooling, but falling when the cooling is rapid, and still more when the alloys are cold-rolled or drawn. It is thus possible, by quenching a rod and then drawing it to the limit, to reduce the value of the expansibility by an amount $1{\cdot}5 \times 10^{-6}$ below that corresponding to the natural state, and thus to confer upon it a negative value. Then, when this condition is attained, a reheating of some hours at 100°C., for example, brings the value to the immediate neighbourhood of zero. Taking advantage of these facts a method has been elaborated which has rendered possible the production of kilometre-lengths of invar wire whose expansion with temperature can only be detected by the most precise measurements. This result is of great practical importance, especially to geodesy.

It was a matter of some interest to follow the expansion of the alloys over a more extensive range of temperature than that permitted by the comparator, even at the sacrifice of some of the accuracy. In 1896, I was able to extend the measurements as far as 220°C., and to determine the manner in which β varies. The appearance of the curves immediately

suggested a generalisation, which allowed of the prediction of phenomena before suitable methods for their study had been elaborated.

Even within the range of measurements by the comparator, it is evident from an examination of Figs. 1 and 5 that the high values of β correspond to the region of temperature within which the alloys pass gradually, on cooling, into the magnetic state. I thought that the same might hold in the case of those alloys whose transformation commences at a temperature above the limit which my experiments had attained; and I anticipated that, at an equivalent distance from the commencement of the appearance of magnetic properties, different alloys would be in approximately analogous states.

Taking, then, the values of the expansibilities over the particular range of my experiments, I plotted them against nickel content, and found that the resulting curve is of similar shape to that representing the expansibility of one and the same alloy through all the transformations which it undergoes over a large interval of temperature. Thus was obtained the "rule of corresponding states" for the nickel-steels. Employed with care—for it is not a precise law—it has proved of the greatest utility.

In order to apply this rule, we may start with the value of β appropriate to the alloys containing 70 per cent. Ni, at which percentage the temperature of magnetic transformation attains its maximum value; and pass progressively through the alloys of gradually diminishing nickel content, down to that containing from 26 per cent. to 27 per cent. Ni, for which the magnetic transformation occurs at the ordinary temperature. (*see* Fig. 1). On plotting the values of β at ordinary temperature against nickel content a curve is obtained, which proves to be none other than that found on plotting the value of β for a single alloy against temperature over a considerable range.

The value of β, to begin with, is positive; and the corresponding portion of the expansion curve (*AB* of Fig. 6) is slightly concave upwards. We then enter the region of negative values of β, and the curve bends downwards (*BC*) thus passing into the region of low coefficient of expansion (*CD*). β then recrosses the zero line, and becomes rapidly positive; its value increases, reaches a maximum, then falls to the region of normal values, the coefficient of expansion remaining, however, very high (*EF*).

The series of values thus deduced directly from β is found to agree closely with that obtained on examination of the corresponding values of α. Thus, from a comparison of the two diagrams, the unity derived from the consideration of corresponding states is seen to be perfect.

The progress of technical metrology has, within the last few years, made it possible to examine the expansion of a great

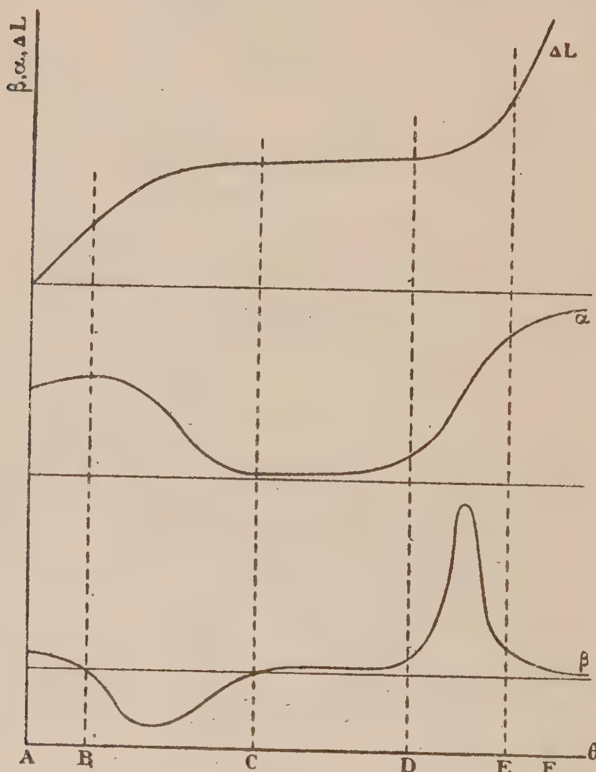


FIG. 6.

This diagram represents the expansion ΔL, the true coefficient of expansion α, and the quadratic coefficient β, of a reversible nickel-steel, at different temperatures.

number of nickel-steels over a range of a thousand degrees. M. Chevenard especially has made an extensive series of investigations, which have resulted in the verification of the general forecasts. They have at the same time shown that the expansion curve of the alloys gradually changes character :

with increasing nickel content the range of temperature over which the volume remains practically constant becomes smaller and smaller ; the expansion curve concurrently altering in slope so that in the alloys of large nickel content no very low value of the coefficient is to be found anywhere within the total temperature range.

Effect of Additions.—A glance at the curve of Fig. 4 shows that in the anomaly under consideration the rule of mixtures, flagrantly at fault in regard to the two constituent elements iron and nickel, is no better respected when we pass to the mixture of invar with one or other of its components. Except in the immediate neighbourhood of the minimum, the coefficient of expansion rises much more rapidly, whether on addition of nickel or of iron, than according to the rule of mixtures.

It would, therefore, appear probable that the same would apply when additions other than those of an excess of its principal constituents are made to invar.

In this connection the study of changes of expansibility on addition of other components is of considerable practical interest, since it may be desired to modify the general properties of alloys possessing the anomaly, in order to adapt them to particular classes of application. In preparing castings, for example, one would increase the manganese content ; or, again, if it were desirable to raise the elastic limit, one would add chromium or carbon.

I have carried the study of these alloys up to the utmost practicable limit of addition. Anticipations have been confirmed, in the sense that the minimum expansibility is always found to be raised ; but, at the same time, it has been found that the minimum abscissa travels along the axis, either towards the iron or towards the nickel.

The two diagrams of Figs. 7 and 8 permit of following the displacement of the minimum, both as regards abscissæ and ordinates. The rest of the curve rises as a whole, so as gradually to obliterate the depression representing the anomaly. But, on account of the displacement in abscissæ, in the three cases in which this is very rapid (Mn, C, and Cu), there occurs, to the right of the minimum for Mn, to the left for C or Cu, a narrow range of contents within which the ternary alloys may have a coefficient of expansion slightly less than that of the binary alloys.

It is just as well to remark that the diagrams are drawn in one respect arbitrarily. They are given in terms of nickel content as abscissæ; if, however, they are referred to iron, the

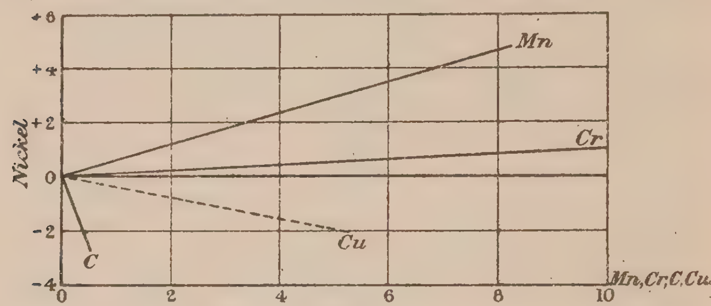


FIG. 7.

Displacement of the abscissa of the minimum of expansibility of the nickel-steels as a function of the proportions of a third constituent. [Abscissæ denote percentage of the additional constituent (Mn, Cr, Cu, C); ordinates denote the changes in percentage of nickel content, starting from the content corresponding to the minimum in the typical alloys.]

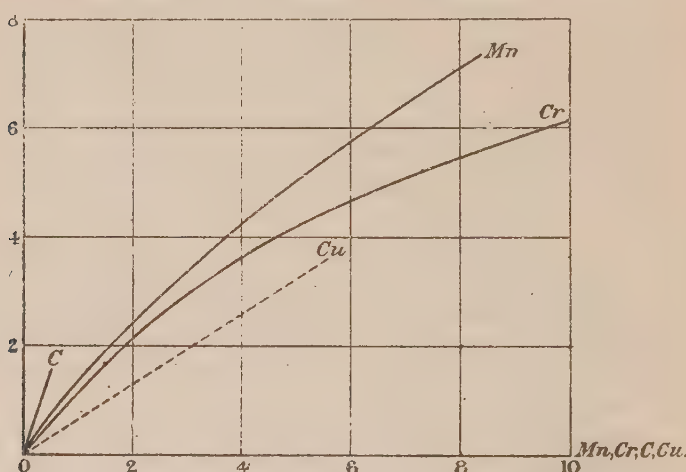


FIG. 8.

Change in the minimum expansibility of the nickel-steels when a third constituent is added. (Abscissæ denote percentages of the added element; ordinates denote the changes in $10^6\alpha_{20}$, starting from the minimum value appropriate to the typical alloys.)

abscissæ would be modified by an amount equal to that of the addition considered. Take, for instance, the binary alloy containing 36 per cent. of Ni and 64 per cent. of Fe; with 8

per cent. of Mn the alloy having the same abscissæ in nickel-content would have, referred to iron, an abscissa of 56.

A diagram drawn according to the method advocated by Guthrie avoids this indefiniteness. The basis of this diagram, as is well known, lies in the property of the equilateral triangle, according to which the sum of the distances of any point in the interior from the three sides is constant, being equal to the height of the triangle. The distance from each of the three sides represents the percentage of the corresponding metal, whilst the ordinate erected perpendicular to the plane of the triangle represents the numerical value of the property under consideration. (At this point the Lecturer showed the Guthrie diagram relating to the ternary alloys of Fe, Ni and Mn, and expressed his pleasure in paying homage to the memory of the founder of the Physical Society, under whose auspices the lecture is delivered).

Elastic Properties.

The Elastic Modulus.—My researches have related mainly to Young's modulus. I have ascertained that the transformation of the irreversible alloys by cooling, whilst raising the elastic limit to a considerable extent, reduces the modulus by a tenth of its value. In the case of the reversible alloys, the modulus falls concurrently with the coefficient of expansion, and attains a value of about a quarter less than that which would follow from the rule of mixtures. The minimum occurs in the invar region, where the modulus has a value of about $1{\cdot}4 \times 10^{12}$C.G.S.

T mperature-variation of the Modulus.—The anomaly in regard to the elastic properties is very remarkable, whether expressed in terms of Young's modulus or of the modulus of torsion.

The examination of the variation of Young's modulus with temperature was carried out on watches provided with springs of the alloy selected; most of the observations were made, under my direction, by Paul Perret, or by the Société des Fabriques de Spiraux Réunies. From the results obtained over a range of 30 degrees C., interpreted in the light of the rule of corresponding states, I have been able to draw representative diagrams, just as in the case of expansibility. These deductions were verified later by Félix Robin.

In regard to torsion, a rapid examination sufficed to establish the existence of effects of the same order as in the case of

flexure. The investigation was extended by M. Ch. Eug. Guye to the case of invar ; and M. Chevenard for the first time made full detailed measurements.

The period of oscillation of a balance-wheel controlled by a spring depends mainly, so far as temperature is concerned, on the variation of Young's modulus ; the expansion of the spring only would cause the watch to gain with rise of temperature, whilst expansion of the balance-wheel only would cause it to lose to about the same extent. In the case, for example, of a steel spring associated with a balance-wheel of brass, the effects of expansion would practically compensate each other, and the going of the watch would have a temperature-coefficient one-half of the thermo-elastic coefficient of the spring.

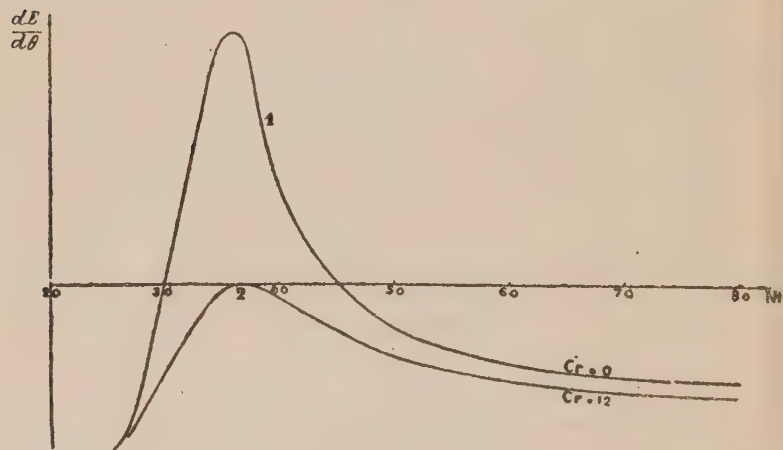


FIG. 9.

Temperate-coefficient of elastic modulus of the nickel-steels as a function of nickel constant. (Curve 1 refers to the pure nickel-iron alloys ; curve 2 to alloys containing added elements equivalent to 12 per cent. of chromium.)

It will always be possible, on making a small correction for the outstanding effect of expansion, to deduce from the rates of the watch the value of the thermo-elastic co-efficient.

Curve 1 of Fig. 9 represents the thermo-elastic coefficients (temperature-coefficients of Young's modulus) in the neighbourhood of 20°C. ; and curve 1 of Fig. 10 the values of the modulus for a particular alloy over a large range of temperature, as deduced from the preceding diagram by application of the rule of corresponding states.

The first diagram is striking on account of its similarity to that of the expansibility. It is evident that if one were to replace the coefficient of variation of Young's modulus by that of expansion, and shear one of the curves parallel to the axis of ordinates, the curves would practically coincide. The correspondence is so exact that it has proved possible, as we shall presently see, to deduce the existence of certain peculiarities in the thermo-elastic properties of the nickel-steels when given their coefficients of expansion.

The main fact disclosed by the curves of the thermo-elastic coefficient is as follows: Between two limits of content at a

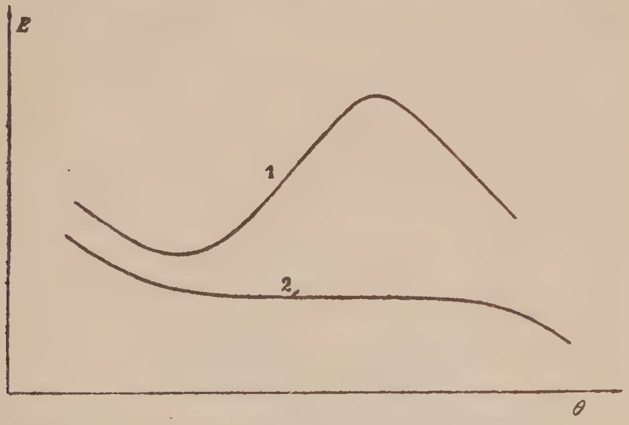


FIG. 10.

The elastic modulus (E) of a nickel-steel as a function of temperature (θ). (Curve I relates to the pure iron-nickel alloy; curve 2 shows the effect of the addition of 12 per cent. of chromium.)

given temperature (or between two limits of temperature at the same content), the thermo-elastic coefficient possesses a positive value; so that between these limits a strip of the alloy, first put under flexure and then raised in temperature, would tend to straighten itself. This fact was directly verified by Marc Thury in 1897, in the case of a strip of invar. The value of the thermo-elastic coefficient in this case was $+0{\cdot}46\times10^{-3}$, that is to say, of the same order of magnitude as that of bronze, but with sign reversed.

Just as in the case of the expansibility, the initial part of curve 1 (Fig. 10) corresponds, at ordinary temperature, to alloys of high nickel content. The minimum is associated

with the second intersection with the zero axis (Fig. 9), the rise with the invar region, and the maximum with the first intersection.

The existence of both the maximum and the minimum of the modulus have given rise to numerous applications wherever there was need of a feebly variable modulus. But in view of the requirements of technical metrology it was deemed desirable to search for a more satisfactory solution.

In order to do this without undue work of a preliminary nature, I made use, as I have said, of the similarity of the curves of expansibility and thermo-elastic coefficient. The diminution in the anomaly found in the first case should find its equivalent in the second, to the extent that having drawn the diagrams of expansibility of the ternary alloys, it should be possible to fix very approximately the proportions of additions conducing to the realisation of a true *elinvar* (or alloy of invariable elasticity). The metallurgical problem was thus reduced to that of discovering the percentages of added metals which, while easy to incorporate with the chief components, should furnish alloys possessing, after appropriate treatment, the requisite qualities, including a high elastic limit.

This problem was solved, like all others incurred in the course of my work, at the steel works of the Société de Commentry-Fourchambault et Decazeville, at Imphy. The principal addition is chromium; a little manganese facilitates the forging; carbon raises the elastic limit, especially when accompanied by a hardening metal. A total addition of about 12 per cent. of foreign metals lowers the curve to that of No. 2, Fig. 9, making it tangential to the zero axis. At the same time the rule of corresponding states indicates that the successive values of the modulus for the same alloy will be represented by curve 2 of Fig. 10. The verification of this inference, made over a range of some 30 degrees, has proved quite exact. In the case of elasticity of torsion M. Chevenard has shown that the actual form of the curve agrees with that predicted over a range of nearly 300 degrees.

Progressive and Transitory Variations.

The properties of the nickel steels, so important in metrology, of which a brief exposition has just been given, are, unfortunately, accompanied by a slight degree of instability, which necessitates the use of certain precautions in the appli-

cation of these alloys. The main effects referred to will be exemplified in the case of invar.

A bar of this alloy which has been forged and allowed to cool in air, then maintained at, say, 100°C., experiences at the outset a rapid increase of length, which can be followed for about 100 hours. If now the temperature of the bar is reduced to 50°, it is observed to elongate afresh, and the movement can readily be followed for over 1,000 hours. If, when the variation has become inappreciable, the temperature of the bar is further reduced to 0°, the elongation once more proceeds.

If the bar is now raised successively to 50° and 100°, being maintained at each of these temperatures in turn for a sufficient time, it recovers sensibly the same lengths as at the termination of the first heatings at these temperatures. The variations in question are thus transitory and slowly reversible.

Again, at ordinary temperatures, elongation may be observed to proceed for a practically indefinite time ; this effect is evidently of a different type from those transient and pseudo-reversible changes above described. It might at first seem as if it were to be regarded merely as the continuation of those changes the observation of which, in the preceding case, was broken off by the passage to a lower temperature—that, in fact, it is the same effect going on during the permanent subjection to one uniform temperature. But it seems preferable to distinguish between "transient changes" and this last effect, which we may designate a "progressive change." The two types of variation are in all respects analogous to those which have long been recognised as occurring in glass, and which are manifested in the changes of zero of thermometers.

It may be gathered from the brief account just given that the rate at which transient changes occur increases with rise of temperature. I have found that for each rise of 20 degrees the rate of change is multiplied about seven times ; which indicates that these changes are to be classed as physico-chemical transformations, for which the velocity of reaction increases between two and three times for a rise of temperature of 10 degrees.

The point to be determined—and it is of capital importance in regard to the applications of invar—is the magnitude of these changes. The numbers about to be given will sufficiently indicate this : the elongation on raising the temperature to 100°, after the bar has been recently heated to the forging

temperature and cooled in air, is of the order of 30 parts in a million ; the complementary elongation observed when the bar is allowed to cool gradually to 0° is of the same order of magnitude. The subsequent progressive change attains, at the expiry of two or three years, a rate of about one part in a million per annum ; and after 20 years the rate is about one-third of this. The total progressive change appears to be approximately 20×10^{-6}.

Again, if the bar is cooled or heated in stages, the changes of length may be represented suitably, between 0° and 100°, by a quantity proportional to the square of the temperature reckoned from the ordinary zero. This cannot be regarded as a natural law, but simply as an empirical relation ; it enables us to conclude that as the zero temperature is approached the changes become extremely feeble.

On proceeding beyond 100°, the quadratic relation holds for a while; then an inflexion occurs, and the rate of change gradually slows down, to attain its limit at a temperature slightly above 200°.

A knowledge of these phenomena serves to dictate, in the first place, the mode of treatment of samples of invar intended for precise measurements ; and, in the second place, to assign the value of the small corrections that have to be appplied to the gross length, as calculated from the instantaneous expansion, in order to allow for progressive or transient changes.

This detail should be added : After a bar of invar has been quenched, the first elongation at 100° is almost double of that which a bar cooled down in air would undergo ; but once the bar has been maintained at 100° till it reaches a steady state, all subsequent changes are of the same order, the heating at that temperature having brought the two bars into the same state as regards departure from stability.

The conclusion is that the same correction formulæ are applicable to all bars which have issued from the same casting and been afterwards subjected to different modes of treatment : provided that all alike are subjected to the initial heating for 100 hours at 100°C. Thus if, for example, the task has once been performed of following for a course of years the changes in one bar of a particular melting, the data thus collected will enable one to calculate the extension of any other bar of that melting. Indeed, I possess a curve of the changes observed over a period of 20 years, which applies to a high degree of

approximation to all bars of invar of approximately typical composition.

With regard to transient changes it may be added that, according to the quadratic law of temperature, their extent amounts, between 0° and 20°, to 4 per cent. of the total change between 0° and 100°. Adopting for this last the value of 30 parts in a million, there remains outstanding a quantity hardly more than one in a million.

The consideration of these numerical values shows that, though the variations in the case of typical invar may prohibit its use in the construction of standards of reference, it is quite feasible to calculate for every future instant the length of an invar standard, to a degree of approximation adequately ample for the great majority of its applications. If there were need of confirmation one might, at ever increasing intervals of time, compare the invar standard with a reference standard. In the case of a standard not less than 10 years old, a comparison made every three years, for example, would allow of interpolation with certainty, on the express understanding that the bar has undergone an initial stoving (ageing), by being maintained for 100 hours at 100°, and afterwards allowed to cool very gradually over an interval of three months.

As has been stated, the figures given above by way of example refer to invar of typical composition. The phenomena, however, though of the same general character, are governed by coefficients which vary very rapidly with the nickel content of the alloy. As the proportion of nickel approaches 42 per cent., the variations become nil, and then change sign; after passing a negative maximum they again gradually fall off, and cease to be measurable at 70 per cent. of nickel. The coefficient of expansion of the 42 per cent. alloy is about 7×10^{-6}; advantage has been taken of the value of the coefficient—the lowest yet met with in a stable cheap metal—to prepare a considerable number of standards from this alloy.

The study of the ternary alloys has recently led me to the discovery of certain relations, some of which had already been approximately foreseen. Thus the addition of manganese or of chromium lowers the instability for a given content of nickel; there is, however, nothing surprising in this, since the anomaly of expansion diminishes at the same time. It is, however, remarkable that a state of complete stability is reached well before the point at which the anomaly disappears.

The presence of carbon, on the contrary, increases the instability. The effect of variation is such as would make it appear very probable from my experiments, that the instability may be attributed entirely to the presence of carbon. Experiments on this point are, however, lengthy and delicate: they are in full swing, and some months must elapse before it is possible to give a full account of them. But there is little doubt in my mind that they will end in the realisation of an alloy of very low expansibility which will be at the same time absolutely stable.

Outline of Theory.

There is as yet no complete and final theory of the phenomena exhibited by the nickel-steels; it is, however, possible to present a tentative sketch throwing light on the main causes of these phenomena.

Iron and nickel can exist in different states separated by allotropic transformations. In the case of nickel, there is only one transformation known, and that not of large extent. It manifests itself at 360° by a sudden lowering of the magnetic susceptibility, a very slight change of volume, and a bend in the expansion curve, the curvature of which falls to about half its value.

Iron, on the contrary, suffers changes of far greater importance, resulting in the alpha, beta, gamma and delta states. The last of these, discoverd by Curie at a temperature above 1,300°, is of no consequence so far as concerns the properties of the alloys of iron at ordinary temperatures. The beta condition disappears as such in alloys containing a fairly high proportion of a second constituent. Moreover, metallurgists are not in unanimous agreement even as to its separate existence, some considering that it is merely a solution of the other two states in each other.

We may dispense, therefore, with both the beta and the delta states, and speak of the reciprocal passage between the alpha and the gamma states, with this reservation, that in the case of alloys containing small quantities of added metals, this passage occurs with a brief intermediary existence in the beta state.

In the pure metal gamma iron exists unaccompanied at all temperatures above 890°. Up to that point iron expands according to a function affected by an important quadratic term (AB, Fig. 11); at that point it undergoes a sudden contraction (BC), and then resumes its expansion with a greater

coefficient, of value 23×10^{-6}, but with smaller curvature (CD). The linear contraction in the ascending transformation is about 3/1,000. The crystalline form, cubic in the alpha state, becomes octahedral in the gamma state. In this last state the metal is feebly magnetic in the Curie sense (and no longer ferro-magnetic).

Let there be added to the iron a metal with which it forms a solid solution ; according to the principle of cryoscopy, the transformation will be lowered in the scale of temperature, and at the same time it will cease to be sudden, being spread over a considerable interval. The abrupt dip is thus con-

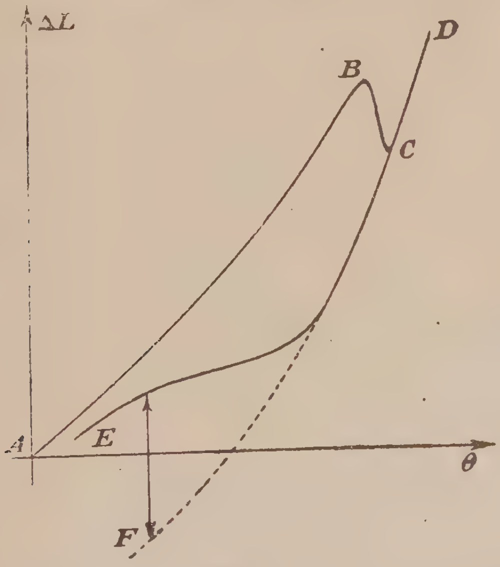


Fig. 11.

Expansion curves of pure iron and of a solid solution.

verted into a more gradual curve CE, at each point of which the change of volume accompanying the transformation is superposed on the ordinary expansion. And, the two phenomena being of opposite signs, the result is the negative anomaly revealed by experiment.

This theory, applied in detail, gives an adequate explanation of reversible transformation. In the case of irreversible changes another factor intervenes. The intersection of the curves of transformation, discovered by M. Dumas, has been

followed out to the fullest extent by M. Chevenard, who has shown that as the nickel content increases, the irreversible transformation diminishes in degree, whilst at the same time the temperature of transformation falls, the transformation vanishing in the alloy of 34·4 per cent. Ni, corresponding to the formula Fe_2Ni. M. Chevenard has thus been led to attribute an important rôle in irreversible transformations to this definite combination which, it may be fortuitously, corresponds very closely to the maximum of the anomaly.

The elastic anomaly may be deduced from that of change of volume by consideration of molecular reactions, which increase considerably as the metal becomes denser. In passing from the alloys to pure iron one might expect a sudden increase in the elastic modulus to occur at the change from the alpha to the gamma state. M. Howe has definitely ascertained that such a change in modulus occurs, though as it takes place at a very high temperature, in a soft condition of the metal, and with changes of volume involving thermal phenomena, one might have been inclined to cast doubt on the convincingness of the demonstration. The elastic properties of the nickel-steels, however, bear witness to its genuineness.

The crystalline change in the irreversible alloys is easy to observe. If a polished fragment of one of these steels is immersed in liquid air it becomes rough, and examination with a lens shows the presence of numerous lancet-shaped crystals of iron. During the change the metal gives a click which is almost musical.

The changes, both transient and progressive, of the nickel-steels appeared to me as an almost necessary consequence of the transformations which occur in the solid solution iron-nickel; this taking up at the outset, when placed at any definite temperature, a condition of approximate equilibrium, changing slowly into one of perfect equilibrium. My recent researches, however, prove that this is not the case. The pure alloys of iron and nickel attain their equilibrium instantaneously; on the contrary, combinations containing carbon—and it is very probable that the same applies to cementite, Fe_3 C, which can exist in two distinct states, separated by a transformation with change of volume—arrive only slowly at their state of equilibrium.

This is, however, beside the main point; a theory relating to the pure nickel-steels is not called upon to include an explanation of this matter.

Applications.

The very remarkable properties of the nickel-steels allow of numerous applications, based either on the anomaly of their expansion, or on that of their elastic properties. In the two domains of measurement of length and of time these applications perhaps call for elucidatory treatment. Only cursory mention need, however, be made of two widely differing applications of these alloys in association with glass, namely, to technical optics and to the manufacture of incandescent lamps.

A region is encountered on the expansibility curve at a nickel content close to 45 per cent. Ni, within which the successive alloys cover the same range of coefficient as the different kinds of glass in general use.* These alloys serve to provide firm mountings for optical apparatus, and leading-in wires for incandescent lamps. In the last case, the wire, if previously freed by long heating *in vacuo* from gas, which is present on considerable quantity, readily welds with the glass. For this purpose it has almost completely replaced platinum, which is thereby set free for other applications. The saving already effected in this way amounts to some millions of pounds.

The Measurement of Length.—The most important of the applications of invar to the measurement of length is the determination of geodesic bases.

During the first three-quarters of the nineteenth century unceasing efforts were made to perfect this measurement regardless of the difficulties and the expense which this entailed. Another tendency then set in, that of simplifying methods, and of making use of longer and more numerous bases. Codifying and systematising diverse processes, M. Edw. Jaderin worked out, about the year 1890, a method consisting in the employment of wires stretched by dynamometers, whereby the respective distances between fiduciary marks mounted on tripods interspersed along the base-line were determined.

But at the time when M. Jaderin first applied his procedure only metals of considerable expansibility were available ; and the precise measurement of length consequently involved that of temperature. The uncertainty as to the equality of temperature of a thermometer and of a wire freely exposed to air

* In the descending part of the curve (at a content of about 29 per cent. Ni) this coefficient is the same at ordinary temperature, but owing to the high value of β it rises rapidly as the temperature rises.

suggested to him the idea of adapting to his procedure the method of Borda and Lavoisier : namely, the use of a bimetallic standard in the form of two wires, of steel and brass respectively, successively employed to measure the length of each lap or element of length of the base, the difference serving to indicate the temperature.

On learning of the existence of invar, M. Jaderin, who was at the time preparing to measure bases in connection with the Suedo-Russian Expedition to Spitzberg, requested me to place the necessary wire at his disposal. I was already occupied with this matter, and had caused wire suitable for base line measurements to be manufactured at the steel works at Imphy; this had actually been used in preliminary trials. The effect of wire-drawing on the expansibility was already partially known, and we were consequently able to supply the Mission with wire practically free from temperature-coefficient.

The experiments at Spitzberg, carried out in 1899, proved remarkably successful. A discussion on the question took place at the International Geodesic Conference at Paris in 1900, at which these results, as well as the systematic experiments carried out by M. Benoit and myself at the Bureau International, were discussed. As a consequence a request was made to the International Committee of Weights and Measures that the Bureau should continue its investigations, and undertake the determination of data relating to wires intended for geodesic measurements.

A mural base was immediately erected, and has served for these investigations, which have been continued without a break since 1901.

The number of observations carried out in the course of these 20 years now totals some hundreds of thousands. Our work has been richly rewarded ; it has made it possible to define the conditions under which wires, specially treated, and whose length has been determined at the Bureau or other standardising laboratory, may be wound on drums, transferred to any required spot, unwound, and employed without any modification of their length.

In order to put to fullest advantage the new precision securable by the use of wires of invar, we have designed a complete set of apparatus, made by Carpentier, by means of which one can be certain that in the measurement of a lap, normally of 24 metres, no error exceeding one hundred-thousandth of the length shall be committed. In accordance with

the principle of the addition of errors, it follows that a length of 100 laps will never be in error by one part in a million.

Numerous base-line measurements have been conducted in recent years with the aid of invar wire. Checks are obtained, either by making the measurement first one way and then the other, or by successive measurements made with different wires; it has thus been shown that an accuracy of one in a million is quite normal. As the apparatus is easily installed, it is now possible to work over a terrain which would have been quite unsuited to measurements with the older form of apparatus, including, as it did, rigid scales and microscopes. Furthermore, as it is quickly set up, and the laps are longer, and the apparatus is light and readily transported, the staff

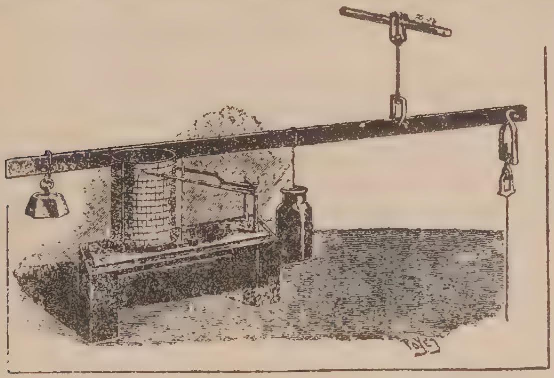


FIG. 12.
Recording apparatus used on the Eiffel Tower.

of observers and assistants can be reduced to one-fifth, and the time required to one-tenth. The total cost is thereby reduced to one-fiftieth of its former amount.

This economy is reflected in the measurement of angles, since the multiplication of base lines, and their greater length, reduce in considerable proportion the consequences of errors committed in angular measurements; it is thus possible to mitigate the excessive precautions to obtain accuracy which were necessary in the older geodesy.

Vertical Movements of the Eiffel Tower.—When this celebrated tower was erected, the French Service Geographique was concerned with the determination of the amplitude of its oscillatory movement, whether caused by wind or by inequalities of temperature occasioned by solar radiation; but

for want of an appropriate method no attempt was then made to study the vertical movements.

By the use of invar wire the problem was solved quite simply. A wire of this material was rigidly attached to the ground at its lower end, and connected at the other to a lever mounted on the second platform of the tower, and which actuated a registering mechanism (Fig. 12). A damper attached to the lever allowed this to return slowly to the position determined by the wire in its straight position, whenever the wire was momentarily curved by a puff of wind. Experiment showed that however violent the air movements, intervals of calm occurred from time to time, of sufficient duration to allow the lever to come to rest. Its limiting positions thus marked the true movements of the tower, whilst a jagged edge, resembling

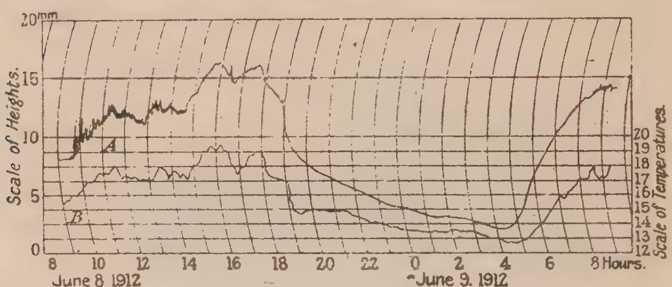


Fig. 13.

The Eiffel Tower. A diurnal record. (A shower of rain at 19·00 has caused a sudden fall in temperature.)

a mop of hair commencing at the envelope of the curve, indicated the effect of the wind.

Examples of the kinds of movement that were observed are shown in Figs. 13 and 14. The first of these was obtained with a 24-hour period of revolution of the recording drum; the second with a period of a week. The lower curve in each figure gives the record of temperature.

On inspection of the two series of curves one cannot fail to be struck with the extraordinary correspondence between temperature and displacement, each minute turn in the one finding its counterpart in the other. The Eiffel Tower thus plays the part of a gigantic thermometer, of great sensibility, despite its enormous mass.

The problem solved by invar wire in the case of the tower is a general one. The above application served to show that it

was easy to record the movements of a metal structure, whether occasioned by variations of temperature or, as in the case of a bridge carrying traffic, by variations of load.

From the two series of curves the coefficient of expansion of the tower may be calculated.

Let us imagine the tower reduced in scale to a millionth of its actual size. In virtue of its symmetry of structure it would constitute a crystal of the size of a pin's head. Your President would, thanks to the admirable methods which he has devised, and which he has so beautifully applied, be able to determine with ease the coefficient of expansion of a crystal of this minute size. And I must confess, speaking as a metrologist, that I should be far prouder of having determined the expansibility of a pin's head than that of the Eiffel Tower.

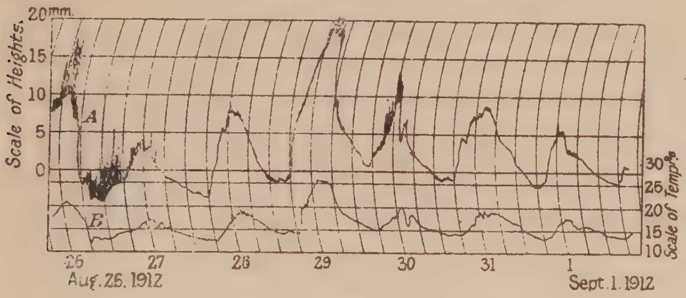


FIG. 14.
The Eiffel Tower. A weekly record. (A stormy week.)

The Measurement of Time.

Clocks.—Of the different methods devised to compensate the pendulum of a clock on account of temperature, by far the most commonly used during the past century was that of Graham, in which the upward expansion of mercury contained in a vessel attached to the rod was made to counteract the downward expansion of the rod itself.

In invar, one finds, according to treatment, all values of the coefficient of expansion between two narrow limits near zero; and having first chosen the material for the rod, it is easy to select that for the bob, provided with screw adjustment, so as to correct by its upward expansion for the elongation of the rod. The advantages of this are manifold. Apart from the presence of a liquid in an oscillating system, Graham's pendulum has the slight defect that it is only truly compen-

sated if the temperature is the same from top to bottom of the clock-case, a condition only sufficiently satisfied in special circumstances.

It may be remarked that for this particular application, the instability of invar is of little consequence, since the going of a clock has in any case to be determined from time to time, in order that any slow variation of its diurnal rate may be accounted for. After the lapse of a few years this variation will have fallen to a few hundredths of a second per annum.

Watches.—A watch provided with a steel spring and a brass balance-wheel loses about 11 seconds per degree rise of temperature per day; and, as we have seen, practically the whole of this variation of rate is due to the change of Young's modulus in the metal composing the spring.

Ferdinand Berthoud's method of correcting for this variation consists in modifying the effective length of the spring automatically by the action of a bimetallic strip; whilst Arnold advocated the employment of a balance-wheel constructed of a circular bimetallic strip, whose moment of inertia alters in the same sense and in the same mean proportion as the modulus of the spring. If for the steel spring one of nickel-steel is substituted, of composition such that the maximum or the minimum of the modulus occurs at ordinary temperatures, it is possible for the watch, although provided with a balance-wheel of one metal only, to possess the same rate at two temperatures situated on either side of the mean of that of the surroundings, say, at 0° and 30°; it would attain a maximum rate, either of losing or gaining, at 15°. The slope of the curve is such that the maximum of the modulus (the minimum point occurs in alloys of low elastic limit) corresponds to a gain of 20 to 25 seconds per day compared with the rate at the extreme temperatures—an amount 12 to 15 times less than that observed when an ordinary steel spring is used. This improvement is fully appreciated by watch manufacturers, as may be judged by the fact that close on three million watches are provided annually with these compensatory springs.

Chronometers.—The balance-wheel of Arnold does not completely compensate the effect of temperature on the rate of a watch. Ferdinand Berthoud in 1775 pointed out a secondary error in his own compensation, and this was rediscovered for Arnold's by Dent in 1832; it consists in the fact that a watch provided with a steel spring, and compensated by a

balance-wheel compounded of steel and brass, gains from two to three seconds a day at 15°, assuming the rate of going to be correct at 0° and at 30°.

Considerable efforts have been expended, especially in England, in the endeavour to eliminate the "Dent error." It will be shown that the appropriate use of a nickel-steel allows this result to be simply attained.

The explanation of the Dent error lies in the following fact: The elastic modulus of the spring varies with temperature according to a curve *OH* of decided curvature (Fig. 15A). The

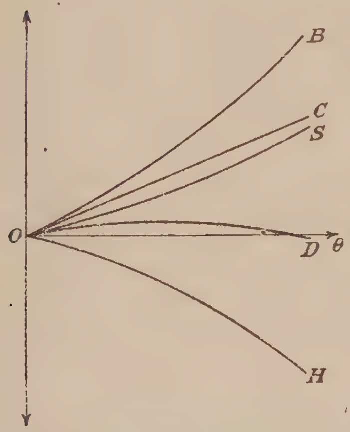


FIG. 15A.
Diagram illustrating the Dent error.

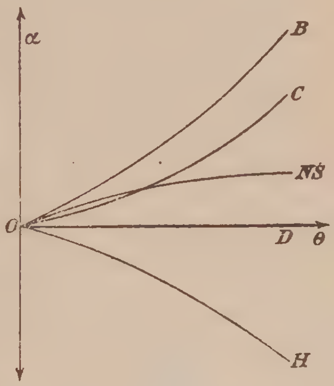


FIG. 15B.
Diagram illustrating the elimination of the Dent error.

action of the balance-wheel, again, is determined by the difference of the expansibilities of brass and steel, *OB* and *OS*, which are expressed by functions in which the coefficients β are very nearly the same. Their difference will, therefore, be represented by a straight line *OC*, and the algebraic sum of *OH* and *OC* will leave a quadratic residue *OD*.

Let us now substitute for *OS* a curve *ONS* (Fig. 15B) representing the expansion of a nickel-steel for which β is negative. The difference between it and *OB* is a curve *OC*, which can be made symmetrical with *OH*, so that the algebraic sum *OD* is zero at every point.

In 1899 I worked out calculations relating to the construction of such a balance-wheel, the element of low expansibility

being constituted by a steel containing 45 per cent. of nickel; and the same year two Swiss makers of chronometers, M. Nardin and M. Ditisheim, proceeding on the basis of this, succeeded in effecting a compensation which was exact at all temperatures, as was shown by the comparisons made under the direction of Dr. Ad. Hirsch at the observatory of Neuchâtel.

The fact that the expansibility of nickel-steel is sensibly inferior to that of ordinary steel has made it possible to make the strips thicker and shorter, thus reducing decidedly the effects of centrifugal force.

Since the date last mentioned the new balance-wheel has replaced the old one practically everywhere. The records of rate have been improved not only by reason of suppression of the Dent error, but also because, once that error was eliminated, it became feasible to seek for ways of correcting for minor defects in all of the parts of a watch. To-day the best chronometers possess variations of rate only a quarter of those found 20 years ago. At Kew the arrival of the new balance-wheel in a watch of Ditisheim's displaced the record from 92·7 to 94·9. By to-day, at Teddington, the mark 96·9 has been reached by a specimen from the hands of the same maker.

It is, however, not unlikely that this new balance-wheel will soon be equalled, by aid of a method of compensation far simpler. Recent results relating to alloys of constant elastic modulus should lead to the supply of springs no longer in need of correction, primary or secondary. Used in conjunction with balance-wheels of a single metal suitably chosen, they will ensure, in watches provided with them, rates identical at all the ordinary temperatures. It seems hardly conceivable that the mechanism of compensation can be reduced to greater simplicity.

Conclusion.

The results which have been set forth in the course of my lecture lead to a conclusion which can be expressed in the following words: The constant struggle going on between alpha- and gamma-iron in their solid solution with nickel constitutes, in the great diversity of its aspects, one of the happiest phenomena which metallurgy has placed at the disposal of metrologists.

XXXIII. *Convective Cooling and the Theory of Dimensions.* *By* Lewis F. Richardson.

(Communicated by C. Chree, LL.D., F.R.S.)

Received February 27, 1920.

The late Lord Rayleigh in a most instructive article on the "Principle of Similitude" ("Nature," Vol. 95, p. 66, 1915) derives a general functional relation for the rate of loss of heat J from a solid of linear dimensions l immersed in viscous fluid moving, at a considerable distance from the solid, with the velocity V. It will be of interest to extend Rayleigh's result so as to include the case in which the velocity near the hot body is partly or entirely produced by the rising of portions of fluid which have been expanded by the heat, as, for instance, in the hot wire ammeter, or as when a kata thermometer is used in calm or nearly calm air.

The essence of the principle of similitude appears to b that phenomena go their way independently of the units whereby we measure them.

A proposed variation in Rayleigh's treatment of the principle was suggested by Riabouchinsky ("Nature," Vol. 95, p. 591, 1915), but was negatived by both Rayleigh and Larmor ("Nature," Vol. 95, p. 644, 1915).

Among the physical variables which affect the phenomenon are the shape of the body and its orientation. These are supposed to be fixed and the others appear to be all comprised in the following list :—

		Dimensions.			
		Length	Mass.	Time.	Temp
Rate of loss of energy by solid	J	2	1	−3	0
Temperature difference between fluid and surface of solid	$(\theta-\theta_a)$	0	0	0	1
Linear dimension of solid	l	1	0	0	0
Velocity of fluid at a distance from solid	V	1	0	−1	0
Thermal capacity of unit mass of fluid ...	γ	2	0	−2	−1
Density of fluid	ρ	−3	1	0	0
Thermal conductivity of fluid	κ	1	1	−3	−1
(accel. of gravity)×(change of density per degree)$=g\partial\rho/\partial\theta=$	β	−2	1	−2	−1
Viscosity of fluid	μ	−1	1	−1	0

Heat energy is supposed to be expressed in work units wherever it occurs explicitly or implicitly.

In this notation Rayleigh's result is

$$J=\kappa l(\theta-\theta_a)F\left\{\frac{lV\gamma\rho}{\kappa},\quad \frac{\gamma\mu}{\kappa}\right\},$$

where F is an arbitrary function of the two variables $(lV\gamma\rho)/\kappa$ and $\gamma\mu/\kappa$. To obtain a correspondence with the experimental results of L. V. King * we may put in Rayleigh's formula

$$F=\left\{\mathbf{A}\sqrt{\frac{lV\gamma\rho}{\kappa}}+\mathbf{B}+\mathbf{C}\frac{\gamma\mu}{\kappa}\right\}.$$

Then

$$J=(\theta-\theta_a)l\left\{\mathbf{A}\sqrt{lV\gamma\rho\kappa}+\mathbf{B}\kappa+\mathbf{C}\gamma\mu\right\}.$$

The terms in A and B are of the form found experimentally by King, but he also found a term $J\propto(\theta-\theta_a)l^2$, which did not involve the velocity. Now it is impossible to choose Rayleigh's F so as to introduce l and to omit V. Thus a wider examination is indicated.

If we form from the nine variables in the list any functions $X_1, X_2 \ldots X_r$ which are of zero dimensions, then any arbitrary relation between $X_1 \ldots X_r$ would satisfy the requirements of the theory of dimensions. And any observed relationship must be expressible as a relation between variables of zero dimensions. Since the variables in the list involve only *powers* of the units of measurements, no generality is lost by making $X_1 \ldots X_r$ severally products of powers of the initial variables. Let U be any such product with indices at first arbitrary, say

$$U=J^{n_1}\,.\,(\theta-\theta_a)^{n_2}\,.\,l^{n_3}\,.\,V^{n_4}\,.\,\gamma^{n_5}\,.\,\rho^{n_6}\,.\,\kappa^{n_7}\,.\,\beta^{n_8}\,.\,\mu^{n_9}. \quad . \quad (1)$$

Then by independence of unit of length .

$$0=2n_1+n_3+n_4+2n_5-3n_6+n_7-2n_8-n_9 \quad . \quad . \quad (2)$$

by mass

$$0=n_1+n_6+n_7+n_8+n_9 \quad . \quad . \quad (3)$$

by time

$$0=-3n_1+n_4-2n_5-3n_7-2n_8-n_9 \quad . \quad (4)$$

by temperature

$$0=n_2-n_5-n_7-n_8 \quad . \quad . \quad . \quad (5)$$

These four equations allow us to express four of $n_1 \ldots n_9$ in terms of the other five, which then remain arbitrary. When we come to compare with experiment, it will be convenient to have as arbitrary the indices of those quantities which have

* Phil. Trans. A, Vol. 214 (1914), pp. 373-432. See also equations (9 and (10) of the present Paper.

been varied in the course of the experiment, say, those of J, $(\theta-\theta_a)$, l, V. Solve then equations (2), (3), (4), (5) for n_6, n_7, n_8, n_9. The result is

$$\left.\begin{aligned} n_6 &= +3n_1+4n_2-n_3+2n_4 \\ n_7 &= \ \ n_1+3n_2-n_3+n_4-n_5 \\ n_8 &= -n_1-2n_2+n_3-n_4 \\ n_9 &= -4n_1-5n_2+n_3-2n_4+n_5 \end{aligned}\right\} \quad . \quad . \quad . \quad (6)$$

So that U splits up into five factors with arbitrary indices as follows :—

$$U=\left(\frac{J\rho^3\kappa}{\beta\mu^4}\right)^1\times\left(\frac{(\theta-\theta_a)\rho^4\kappa^3}{\beta^2\mu^5}\right)^{n_2}\times\left(\frac{l\beta\mu}{\rho\kappa}\right)^{n_3}\times\left(\frac{V\rho^2\kappa}{\beta\mu^2}\right)^{n_4}\times\left(\frac{\gamma\mu}{\kappa}\right)^{n_5}. \quad (7)$$

Each of these factors is, therefore, of zero dimensions. Omitting the indices let us name the factors as follows :—

$$\left.\begin{aligned} &\frac{J\rho^3\kappa}{\beta\mu^4}=X_1\,; \quad \frac{(\theta-\theta_a)\rho^4\kappa^3}{\beta^2\mu^5}=X_2 \\ &\frac{l\beta\mu}{\rho\kappa}=X_3\,; \quad \frac{V\rho^2\kappa}{\beta\mu^2}=X_4\,; \quad \frac{\gamma\mu}{\kappa}=X_5 \end{aligned}\right\} \quad . \quad . \quad . \quad (8)$$

By assigning suitable values to $n_1 \ldots n_5$ we can give to $n_6 \ldots n_9$ any values which are consistent with zero dimensions for U. Therefore, any function of our original nine variables, which is of zero dimensions, can be expressed as a function of $X_1 \ldots X_5$ as defined in (8).

Of recent experimental researches on cooling there is Prof. L. V. King's very thorough study of the loss of heat from fine platinum wires ("Phil. Trans." A., Vol. 214 (1914), pp. 373-432) and Dr. Leonard Hill's calibration of his kata-thermometer. L. V. King exposed his wires broadside to the current and he expresses his experimental results in a formula, which, if we neglect two very small terms, takes the form

$$J=(\theta-\theta_a)l\{A\sqrt{Vr}+B+Cr\} \quad . \quad . \quad . \quad . \quad (9)$$

Here r is the radius of the wire and A, B, C do not depend appreciably on l, r, $(\theta-\theta_a)$ or V. If, to give the wire a constant *shape*, we make l proportional to r, then (9) takes the form

$$J=(\theta-\theta_a)[A'V^{\frac{1}{2}}l^{\frac{3}{2}}+Bl+C'l^2] \quad . \quad . \quad . \quad . \quad (10)$$

where A', C' are new constants.

This result must be expressible in terms of $X_1 \ldots X_5$ without other variables.

As J occurs only in X_1, and $(\theta-\theta_a)$ only in X_2, and l only in X_3, and V only in X_4, it follows that equation (10) must be of the form

$$X_1=X_2\{A''X_4^{\frac{1}{2}} \,.\, X_3^{\frac{3}{2}}+B''X_3+C''X_3^2\} \quad . \quad . \quad . \quad (11)$$

And as X_5 is the only one of $X_1 \ldots X_5$ which King did not vary experimentally, it follows the A'', B'', C'' may depend on X_5, but, if not, then they are dependent only on the shape of the solid and on its orientation to the flow.

Equation (11) simplifies to

$$J=(\theta-\theta_a)l\left[A''\sqrt{\frac{Vl\rho\kappa^2}{\mu}}+B''\kappa+C''l\frac{\beta\mu}{\rho}\right] \quad . \quad . \quad . \quad . \quad (12)$$

King had arrived at the κ in the middle term in the bracket theoretically and had obtained a numerical value agreeing with experiment to 5 per cent. For the first term in the bracket King deduced, from a hydrodynamical theory, the form $\sqrt{Vl\rho\gamma\kappa}$, which we obtain if, remembering that A'' is an unknown function of X_5, we put $A''=X_5^{\frac{1}{2}}A'''$. Thus finally

$$J=(\theta-\theta_a)l\left[A'''\sqrt{Vl\rho\gamma\kappa}+B''\kappa+C''l \,.\, \frac{\beta\mu}{\rho}\right], \quad . \quad (13)$$

where A''', B'', C'' may be functions of X_5, of shape and of orientation.

The new result now brought to light is that the last term in the bracket in equation (13) is of the form $\left(l\,\frac{\beta\mu}{\rho}\right)\times$ (a function of X_5). This X_5, be it noted, is much the same for all gases and does not depend on their density over a wide range (*see* Jeans' "Dynamical Theory of Gases," 2nd Ed., p. 317).

Dr. Leonard Hill and his assistants (Medical Research Committee Special Report No. 32) in investigating his "katathermometer" also found that the rate of loss of heat by convection alone varied, at least roughly, as the square root of the pressure.

Hill, Griffith and Flack ("Phil. Trans." Roy. Soc., B.207, p. 197) give a diagram showing a simple proportionality between excess of temperature and rate of loss of heat from convection and radiation jointly; but, as they point out, the linearity is due to the "curvature" of Stefan's radiation law compensating, in this case, the "curvature" due to convection alone. The two curvatures are of opposite sign

because these experimenters increased the excess of temperature by cooling the enclosure, instead of by heating the katathermometer. Indeed by measuring Hill's graphs (Figs. 4A and 5, Report *loc. cit.*), I find, quite roughly, that the index σ approaches 1·2.

Although equation (13) holds over a wide range, yet when we remember the complexities of the transition from turbulent to non-turbulent flow, as exhibited by Stanton and Pannell, we must doubt its general validity. *If wider experimental researches give relations between J, $(\theta-\theta_a)$, l, and V, different from those found by King for fine wires, then $X_1 \ldots X_5$ will provide a ready means of generalising the result.*

This inquiry arose at Benson Observatory in connection with the lag of thermometers intended for exploring the upper air. It is communicated by permission of the Director of the Meteorological Office.

I am indebted to my wife for solving equations, to Mr. W. H. Dines for reading the manuscript, and to the Referee of the Physical Society for a number of references.

ABSTRACT.

The Paper consists of an application of the "Principle of Similitude" (Rayleigh, "Nature," Vol. 95, p. 66, 1915) to the loss of heat from hot wires, thermometer bulbs, &c. An equation is obtained which does not involve the dimensions of the bodies and which agrees with the formulæ of King for fine wires and of Hill for thermometer bulbs.

DISCUSSION.

Mr. F. J. W. Whipple said he presumed the author took both forced and free convection into account. There were many meteoric problems to which the application of the theory would be of value. Aitken had compared the radiation and convection losses of bodies by exposing them to the sun and measuring their rise in temperature. Using a series of blackened cubes, he had obtained values for the convection which agreed roughly with Barratt's square root law, and would no doubt also fit in with the results of this Paper.

Mr. Gosling pointed out that formula No. 13 appeared to make the convection loss increase with the viscosity of the gas. Langmuir had arrived at the opposite conclusion.

Mr. J. Guild asked if in applying the theory of dimensions to the heat loss from bodies of different sizes, the considerations were not vitiated by the fact that the size and mean free path of the gas molecules were not varying with the other parts of the apparatus.

The Author, in reply, said that the effect mentioned by Mr. Gosling was apparent only. Actually from the nature of the other terms involved, the equation gave reduced values of the convection loss as the viscosity increased. As regards the invariant size of the gas molecules, mean free path, &c., probably the equations cease to hold when any of the dimensions become comparable with the mean free path of the gas.

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